Hierarchical power allocation games

Mehdi Bennis

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Chapter 1

Hierarchical Power Allocation Games

Mehdi Bennis, Centre for Wireless Communications, University of Oulu, Finland
Samson Lasaulce, Alcatel-Lucent Chair in Flexible Radio, SUPELEC, France
Merouane Debbah, Alcatel-Lucent Chair in Flexible Radio, SUPELEC, France

In this chapter, we look at the key issue of power allocation (PA) where the target is the competitive maximization of the information throughput sustained by each link over the network. More specifically, we focus on the concept of hierarchy which exists between different radios/systems sharing the same resources. This paradigm therefore requires a new design and framework aiming towards distributed approaches. For this reason, Game theory (GT) is used as a tool to model the interaction between several players and predict the outcome of the PA game. In particular, a special branch called hierarchical games is adopted wherein radios interact to maximize their respective payoffs following a leader-follower approach. The presented results corroborate the fact that the overall efficiency of the network is thereby improved.
CHAPTER 1. HIERARCHICAL POWER ALLOCATION GAMES

1.1 Introduction

Over the course of the last couple of years, we have witnessed a new paradigm wherein the radio spectrum allocation has gone from static to flexible. Due to the ever increasing needs for more stringent data rates and spectrum under-utilization [12], spectrum sharing [21], [31], [13] and [17] has emerged as a new way to improve the spectral efficiency of radio systems. Two approaches basically exist under the radio resource sharing umbrella: the underlaying and the overlaying [39], [40], [41] scenario. In the first case, primary and secondary radio devices coexist as long as the interference temperature is satisfied. In the second case, secondary radios constantly sense the environment (i.e., spectrum holes) left vacant by primary radios where spectrum holes can be reused by secondary systems provided that they might be required by the primary radios at any time.

In this chapter, we examine the problem of power allocation with a main emphasis on the concept of hierarchy existing between radios. This concept naturally arises in multiple practical situations: a) when primary and secondary systems share the spectrum, b) when user have access to the medium in an asynchronous manner, c) when operators deploy their networks at different times and d) when some nodes have more power than others such as the base station. Within the realm of GT, the hierarchical spectrum sharing problem is naturally modeled using Stackelberg [38] [20] framework motivated by the fact that the non-cooperative Nash equilibrium (N.E) is generally inefficient and non-optimal. As a result, Stackelberg approach provides (in general) better outcomes as compared to the non-cooperative approach. Furthermore, it improves the overall network efficiency hence bridging the gap between the selfish and fully centralized approach.

Clearly, radio devices are very likely to interact upon accessing spectral resources. This interaction happens on both short and long terms. In a non-cooperative approach (i.e., competitive operators) [31], radios behave selfishly by maximizing their payoffs (throughput, revenues, etc). Therefore, these interactions are modeled using Game Theory (GT) [20] a branch of mathematics which has attracted considerable attention for analyzing wireless communication systems. In essence, GT models the strategic interaction between players (radios, transmitters, nodes etc) where an equilibrium point is found and analyzed. GT
encompasses a rich plethora of disciplines such as non-cooperative games [29], cooperative 
[42] games, coalitional games [43], static, and dynamic, with complete and incomplete [15] 
information. In this chapter, we specifically focus on hierarchical spectrum sharing games 
in wireless communication systems.

This chapter is organized as follows. In Section 1.2, we recall the fundamental and rele-
vant concepts/notions of game theory used throughout the chapter. First, we introduce some 
game theoretical definitions such as the concept of Nash equilibrium for non-cooperative 
games followed by the Stackelberg approach. Section 1.3 looks at the applications of hier-
archy in power allocation/control problems arising in wireless communication. We finally 
conclude and discuss open problems in Section 1.4.

1.2 Review of game-theoretical concepts

1.2.1 Two important multi-user channel models

Two elementary channel models exist in the literature which are subject to interaction: the 
multiple access channel (MAC) and the interference channel (IFC) [1]. These are defined as 
follows:

- **Multiple access channels (MAC):** The multiple access channel consists of $K$ trans-
mitters aiming to communicate with a single receiver using a common channel. If 
$N > 1$ channels are available, then there exists $N$ independent or parallel MACs, where 
transmitters in different MACs do not interfere each other. For instance, this model 
corresponds to the uplink channel in a single-cell multi-carrier cellular system. More-
over, the channel gain from transmitter $i$ to the receiver over channel $n$ is denoted by 
$h_{in}^n$. We assume a block flat-fading channel model such that channel realizations remain 
constant during the transmission of $M$ consecutive symbols. All the channel realiza-
tions, $\forall i = \{1, \ldots, K\}$ and $\forall n = \{1, \ldots, N\}$ are drawn from a Gaussian distribution with 
zero mean and unit variance. The power allocated by transmitter $i$ to channel $n$ is
denoted by $p_i^n$. Each transmitter is power-limited wherein for the $i^{th}$ transmitter, its transmit power cannot exceed $P_{i,\text{max}}$, i.e., $\forall i = \{1, .., K\}, \sum_{n=1}^{N} p_i^n \leq P_{i,\text{max}}$.

The symbol sent by transmitter $i$ over channel $n$ is represented by $x_i^n$. We consider that transmitted symbols $\forall i = \{1, .., K\}$ and $\forall n = \{1, .., N\}$ are random variables with zero mean and unit variance. The noise at the receiver is denoted by $w$ and corresponds to an additive white Gaussian noise (AWGN) process with zero mean and variance $\sigma^2$. In vector notation, the channel realizations are written as $h_i = (h_{i1}^1, h_{i1}^N)$. Using a similar notation, the transmit powers, transmitted symbols, and noise are written as $p_i = (p_{i1}^1, .., p_{iN}^N)$, $x_i = (x_{i1}^1, .., x_{iN}^N)$, and $w_i = (w_{i1}^1, .., w_{iN}^N)$, respectively. Finally, the received signal can be expressed as:

$$y(n) = \sum_{n=1}^{N} \sqrt{h_i(n)} x_i(n) + w_i(n) \quad \text{(1.1)}$$

and the received signal to interference plus noise ratio (SINR) assuming single-user decoding (SUD) on channel $n$ for transmitter $i$, denoted by $\gamma_i^n$ for all $\forall i = \{1, .., K\}$ and for all $\forall n = \{1, .., N\}$ is given by:

$$\gamma_i^n = \frac{p_i^n |h_i^n|^2}{\sum_{j \neq i}^{K} p_j^n |h_j^n|^2 + \sigma^2} \quad \text{(1.2)}$$

- **Interference channel (IFC):** The interference channel [1], [2], [36] consists of a set of $K$ point-to-point links close enough to produce mutual interference due to the co-existence on the same channel. If $N \geq 1$ channels are available, there exists $N$ independent or parallel IFCs, where transmitters in different IFCs do not interfere each other. This topology typically appears in self-organized networks (SON) where nodes communicate in pairs over a set of sub-carriers. To describe the IFC model, we keep the same notation and assumptions presented in the MAC case. The only slight modification is introduced to denote the channel realization from transmitter $i$ to receiver $j$ on channel $n$, denoted by $h_{ij}^n$ with $\forall n = \{1, .., N\}$ and $(i, j) \in \{1, .., K\}^2$. The noise at receiver $i$ over channel $n$ is denoted by $w_i^n$ and $w_i = (w_{i1}^1, .., w_{iN}^N)$. The received signal at receiver
1.2. REVIEW OF GAME-THEORETICAL CONCEPTS

\(r_i(n) = \sum_{n=1}^{N} \sqrt{h_{ji}(n)x_i(n)} + w_i(n)\) \hspace{1cm} (1.3)

and the received signal to interference plus noise ratio (SINR) assuming SUD on channel \(n\) for transmitter \(i\), denoted by \(\gamma_i^n\) for all \(i = \{1, \ldots, K\}\) and for all \(n = \{1, \ldots, N\}\) is given by:

\[\gamma_i^n = \frac{p_i^n h_{ii}^n}{\sum_{j \neq i} p_j^n |h_{ji}^n|^2 + \sigma^2} \hspace{1cm} (1.4)\]

Throughout this chapter and for both network topology types, unless otherwise stated, we assume that all transmitters have perfect channel state information (CSI), i.e., each transmitter knows the channel realizations \(h_i\) for all \(i = \{1, \ldots, K\}\) in the MAC case, and \(h_{ij}\) for all \((i, j) \in \{1, \ldots, K\}^2\) in the IFC case.

1.2.2 Normal form games

As pointed out earlier, game theory provides a suitable mathematical framework to analyze the strategic interaction between different players. In general, a game is presented in a normal form as follows:

**Definition 1 (Normal Form)** [20] A game in normal form is denoted by \(\{K, S, \{u_k\}_{k \in K}\}\) and is composed of three elements:

- a set of players: \(K = \{1, \ldots, K\}\),
- a set of strategy\(^1\) profiles: \(S = S_1 \times \ldots \times S_K\), where \(S_k\) is the strategy set of player \(k\),
- a set of utility functions: the \(k^{th}\) player’s utility function is \(u_k : S \to \mathbb{R}_+\) and is denoted by \(u_k(s_k, s_{-k})\) where \(s_k \in S_k\) and \(s_{-k} = (s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K) \in S_1 \times \ldots \times S_{k-1} \times S_{k+1} \times \ldots \times S_K\).

\(^1\)In the case of no coupling among players strategies, the set \(S\) can be written as the Cartesian product of the strategy set \(S_k\) of each player. In the more general case of coupled constraints, each player aims to restrict its strategy to a subset that depends also on the strategies chosen by the other players.
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In the power allocation game, the set of players includes transmitters, base stations, mobile stations, relays, femto base stations, etc. A player’s $k$ strategy is to transmit with a certain power over a certain channel and the utility function is described in terms of Shannon capacity, throughput, revenue, power, etc.

1.2.3 Non-cooperative games, Nash equilibrium and Stackelberg equilibrium

In non-cooperative games, players behave selfishly not caring about other players’ payoffs. Each player chooses its strategy to maximize its utility function. Players are moreover assumed to be rational and adopt the same selfish behavior. In contrast, in cooperative games, each player aims at maximizing a common and social benefit provided other players do the same. The information required to play games is of high importance. For non-cooperative games, only local information is needed whereas all channel state information are generally needed for cooperative games.

The Nash equilibrium (N.E) is an important concept in the field of game theory wherein an N.E corresponds to a profile of strategies $s^* = (s_1^*, ..., s_K^*)$ for which each player’s strategy $s_i^* \in S$ is an N.E if it satisfies:

$$\forall k \in K \quad \text{and} \quad \forall s_k \in S_K, u_k(s_k^*, s_{-k}^*) \geq u_k(s_k, s_{-k}^*)$$

That is, at the N.E, any unilateral deviation from the strategy profile $s_k$ of player $k$, $\forall k \in K$ will not increase its utility function $u_k$. Hence, at the N.E there does not exist any motivation for a player to deviate from the N.E strategy profile. As players are selfish and decide by themselves their strategy, one question arises: does an N.E lead to an efficient game outcome? To answer this question, the notion of optimality comes into play where pareto-optimality is a measure of optimality of a game defined as:

**Definition 2 (Pareto Optimality)**[20] Let $s = (s_1, ..., s_K)$ and $s'_i = (s'_1, ..., s'_K)$ be two different strategy profiles in $S$. Then, the strategy profile $s$ is pareto-superior to the strategy

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profile $s'$ if:

$$\forall k \in K \quad u_k(s_k, s_{-k}) \geq u_k(s'_k, s'_{-k})$$

(1.6)

with strict inequality for at least one player. Moreover, if there exists no strategy that is Pareto superior to $s_i$, then $s_i$ is Pareto-optimal.

The Nash equilibrium is generally inefficient and non-optimal. However, it is a lower bound in the case of non-cooperation between players. To measure the lack of cooperation, the notion of price of anarchy (POA) is defined:

**Definition 3 (Price of Anarchy (POA))** [20] the POA of the game $\{K, S, \{u_k\}_{k \in K}\}$ is equal to the ratio of the highest value of the social welfare (joint optimization) to the worse N.E of the game:

$$POA = \frac{\max_{s \in S} \sum_{k=1}^{K} u_k(s_k, s_{-k})}{\min_{s^* \in S^{NE}} \sum_{k=1}^{K} u_k(s^*_k, s^*_{-k})}$$

(1.7)

where $S^{NE}$ is set of Nash equilibria of the game.

Finally, an interesting performance metric is the social welfare which corresponds to the average utility of players:

**Definition 4 (Social Welfare)** [27] the social welfare of a game is defined as the sum of the utilities of all players:

$$w = \sum_{i=1}^{K} u_i$$

(1.8)

Power allocation games in which the concept of hierarchy is taken into account are referred to as Stackelberg games, initially introduced by Stackelberg in [38]. In these games, there is an implicit concept of hierarchy upon the set of players. Such hierarchy naturally occurs when users play sequentially. For example, in a 2-level Stackelberg game, the game leader moves first and the other players follow and play simultaneously. The game leader perfectly knows the set of strategies and the utilities of the followers who in turn can observe the actions of their leader(s).
A Stackelberg game is solved using the concept of sub-game perfection [20] Nash equilibrium where none of the players have an incentive to deviate in any sub-game. The inefficiency of the Nash equilibrium concept led Selten in [19] to devise the notion of sub-game perfect equilibria in extensive form games. These are defined as equilibria which rely on threats that are really credible. In the first stage of a Stackelberg game, the leader who perfectly knows all the followers set of actions and utility functions, chooses the action that maximizes its benefits considering that each follower will react with the action which maximizes its own benefit as well. Thus, the game leader analyzes all the possible outcomes and picks up the action which maximizes its benefit considering the optimal moves for each player. A formal definition of a Stackelberg game and its equilibrium is given below:

Definition 5 (Stackelberg game) [20] A Stackelberg game is a two-stage game at which one player (leader) moves at the first stage and all the other players (followers) react simultaneously at the second stage.

Definition 6 (Stackelberg Equilibrium) [20] A strategy profile \( (p_{SE}^1, p_{SE}^2, ..., p_{SE}^K) \) is called a Stackelberg Equilibrium if \( p_{SE}^1 \) maximizes the utility of the leader and \( p_{SE}^2, ..., p_{SE}^K \) is the best response of players 2, ..., K to player 1.

1.3 Application to wireless communications

In this section, we investigate direct applications of hierarchical power allocation relevant in wireless communications problems. First, the problem of spectrum leasing between primary and secondary systems is investigated. Second, the concept of hierarchy is investigated in the context of energy-efficiency games. Next, power allocation games in the context of multi-user channels is discussed. Finally, base station location games and the impact of hierarchy are dealt with.
1.3. APPLICATION TO WIRELESS COMMUNICATIONS

1.3.1 Spectrum leasing to secondary systems

In [3], dynamic spectrum sharing is investigated in which primary systems lease the spectrum to secondary systems in exchange for cooperation (PA game). In essence, a primary transmitter $T_P$ wants to send information to its primary receiver $R_P$ either directly with a rate of $R_{dir}$ or alternatively using cooperation from a set of ad-hoc network consisting of a set of $K \in \mathcal{K}$ point-to-point links. For transmission purposes, the primary system divides its data in two parts of $\alpha L$ bit durations and $(1 - \alpha)L$ bit durations, with $0 \leq \alpha \leq 1$. The first $(1 - \alpha)L$ bits are dedicated to a direct transmission from $T_P$ to $R_P$ whereas the second $\alpha L$ bits are again divided into two parts. One part, consisting of $\beta \alpha L$, with $0 \leq \beta \leq 1$, is dedicated to send information from $T_P$ to $R_P$ using the ad-hoc (secondary) network by means of distributed space time coding (DSTC) [14], while the remaining $\alpha(1 - \beta)L$ bits are granted to the secondary network for its own data transmission. The leader maximizes its own utility function while deciding the portion of time-slots $\alpha$, $\beta$ and $k \in \mathcal{K}$ subset of secondary transmitters.

Given the set $\mathcal{K}$ and cooperation parameters $\alpha$ and $\beta$, the rate optimization problem for the leader is given by:

$$\max_{\alpha, \beta, k} R_P(\alpha, \beta, k)$$

$$k \leq \mathcal{K}, \quad 0 \leq \alpha, \beta \leq 1 \quad (1.9)$$

where $R_P(\alpha, \beta, k)$ is defined as:

$$R_P(\alpha, \beta, k) = \begin{cases} 
\min\{(1 - \alpha)R_{PS}(k), \alpha \beta R_{SP}(k)\}, & \alpha > 0 \\
R_{dir}, & \alpha = 0 
\end{cases}$$

where $R_{PS}(k)$ is the achievable rate between the primary transmitter and secondary receiver and $R_{SP}(k)$ is the achievable rate between secondary transmitter and primary receiver. Finally, $R_{dir}$ is the achievable rate with no cooperation ($\alpha = 0$). Once cooperation parameters $\alpha$, $\beta$ are found using (1.9), the secondary system reacts by exploiting the $\alpha(1 - \beta)L$ bits within
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which it can transmit where transmitters play a non-cooperative game \( \langle \mathcal{S}, \mathcal{P}, u_i(p_i, p_{-i}) \rangle \) yielding the Nash equilibrium.

The optimization problem for the secondary transmitters (from the set \( \mathcal{S} \)) is written as:

\[
\max_{p_i} u_i(p_i, p_{-i}) = \max_{p_i} (1 - \beta) \log_2 \left( 1 + \frac{|h_{S,ii}|^2 p_i}{\sigma^2 + \sum_{j=1,j\neq i}^k |h_{S,ij}|^2 p_j} \right) - c.p_i
\]

\[0 \leq p_i \leq P_{\text{max}} \quad (1.10)\]

where \( c \) being the cost per unit transmission energy. Note that Equation (1.10) only depends on \( k, \alpha \) and \( \beta \). Finally, solving (1.10) yields the Nash equilibrium of the non-cooperative game expressed as:

\[
\hat{p}_i = \left( \frac{1 - \beta c}{c} - \frac{\sigma^2}{|h_{S,ii}|^2} - \sum_{j=1,j\neq i}^k \frac{|h_{S,ij}|^2}{|h_{S,ii}|^2} \hat{p}_j \right) + \quad (1.11)
\]

The existence of the N.E is assured by the concavity of the utility function in (1.10) [20]. Moreover, the N.E is unique [3] if the following sufficient conditions are satisfied:

\[
\sum_{j \in \mathcal{S}, j \neq i}^k \frac{|h_{S,ij}|^2}{|h_{S,ii}|^2} < 1 \quad (1.12)
\]

The interaction between the primary and secondary networks is modeled as a strategic Stackelberg game. The leader maximizes his own utility function knowing which in turn affects the secondary network. The Stackelberg equilibrium is solved using backward induction [20] where (1.11) is plugged back in (1.9) after which the optimal \( \alpha^*, \beta^* \) and \( k^* \) are computed maximizing thereby the leader’s payoff. The Stackelberg equilibrium is obtained going back to (1.11) and solving the optimization problem. Finally, it turns out that maximizing the revenue of the primary network results in several tradeoffs. First, if \( \beta \) increases there is more cooperation time at the cost of lessening the cooperation from the secondary network. Moreover, a large value of subset \( k \) may limit the overall rate by reducing the term \((1 - \alpha)R_{SP}(k)\) while at the same time it enhances the term \( \alpha \beta R_{SP}(k, \beta) \) thanks to cooperation.
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In the partial CSI case, the system optimization is solved from a probabilistic standpoint (outage probability-based) as the primary system is unable to know the subset $S$ of secondary ad-hoc networks willing to cooperate. Moreover, with long-term CSI, the set of secondary transmitters able to decode the primary message is a random quantity, and thus the choice of a space-time code-book and the specific codeword to be transmitted by each node in $S$ cannot be done by the primary. To cope with this, randomized DSTC [10] is used. Similarly to the full CSI case, the primary transmitter wishes to transmit at a given target rate $R_P$ with a required bit error probability (BER). If the instantaneous channel fading conditions are such that the requirement on the BER is not satisfied, an outage is declared. The goal of the primary link is to minimize the probability of outage $P_{out}(\alpha, \beta, C_i)$ for fixed $R_P$ and BER with respect to $\alpha$, $\beta$ and the space-time codebook $C_i$.

The outage probability $P_{out}(\alpha, \beta, C_i)$ is defined as the probability that the Signal to Noise Ratio (SNR) on the cooperative link (denoted by $\gamma_{SP}(S, \beta, C_i)$) experiences an instantaneous SNR smaller than the desired threshold where:

$$P_{out}(\alpha, \beta, C_i) = \sum_S P_{PS}(S, \alpha).P_{out}(S, \alpha, \beta, C_i)$$ (1.13)

$P_{PS}(S, \alpha)$ is the probability that the secondary nodes in $S$ are able to decode the primary transmission in the first slot and:

$$P_{out}(S, \alpha, \beta, C_i) = Pr \left[ \gamma_{SP}(S, \alpha, \beta, C_i) < \gamma_{th} \left( \frac{\bar{R}_p}{\alpha \beta R_{STC,i}} \right) \right]$$ (1.14)

is the outage probability of the randomized DSTC in the second slot when nodes in $S$ are active and the orthogonal STC $C_i$ is used. Essentially, the primary link aims at solving the following optimization problem:

$$\min_{\alpha, \beta, C_i} P_{out}(\alpha, \beta, C_i)$$

s.t $C_i \in C, 0 \leq \alpha, \beta \leq 1,$

$$\alpha N_S, \alpha \beta N_S, \alpha \beta N_S/q_i \in N$$ (1.15)
where the outage probability of the primary link, \( P_{out}(\alpha, \beta, C_i) \), is:

\[
P_{out}(\alpha, \beta, C_i) = P_{out, dir}, \quad \alpha > 0
\]

(1.16)

otherwise \( P_{out}(\alpha, \beta, C_i) = P_{out, dir} \) and \( \mathcal{N} \) is the set of integers.

### 1.3.2 Hierarchy in energy-efficient power control games

Unlike works in which the utility function to be optimized is often the Shannon rate, a hierarchy is introduced in [6] in the context of decentralized multiple access channels with energy efficiency being the ultimate goal. More specifically, this approach aims at maximizing the ratio between the number of information transmitted bits without errors, and the transmit power level where the utility function of user \( 1 \leq i \leq K \) is given by:

\[
u_i(p_1, ..., p_K) = \frac{T_i}{p_i} = \frac{R_i f(SINR_i)}{p_i}
\]

(1.17)

where \( f \) is an efficiency function\(^2\) representing the packet success rate, assumed identical for all users and \( R_i \) is the transmission rate of user \( i \). Moreover, when it exists, the non-saturated\(^3\) N.E of the game is given by:

\[
\forall i \in \{1, ..., K\}, \quad p_i^{SUD} = \frac{\sigma^2}{|h_i|^2} \beta^* \mu^{SUD}
\]

(1.18)

where \( \beta^* > 0 \) is the solution of the equation: \( xf'(x) = f(x) \) and \( \mu^{SUD} = \frac{1}{{1-(K-1)\beta^*}} \) is the penalty term due to multiple access interference.

As shown in [6], there is always a unique Nash equilibrium in the game. If \( \mu^{SUD} \geq 0 \) and none of the users’ power constraints is saturated, the equilibrium is given in (1.18) in which only individual CSI is needed at each transmitter. However, if \( \mu^{SUD} < 0 \), or at least one power constraint is saturated, the N.E has to be rewritten by taking into account the fact

\(^2f \) is a sigmoidal function [23].

\(^3\)The maximum transmitter power for each user, denoted by \( P_i^{max} \) is assumed to be sufficiently high for not being reached at the equilibrium.
that some users transmit with their maximum power $P_{i}^{\text{max}}$.

On the other hand, the issue of information knowledge of the game is important. In [6], it is assumed that the BS knows all the channel gains of the users. However, user $i$ only needs to know his own channel gain $h_i$ to compute his own selfish power allocation. Note that [7]-[8] deal with similar problem with a main focus on flat fading channels and multi-carrier CDMA systems, respectively. It is found out that hierarchy not only improves the individual energy efficiency of all users but also ensures the existence of a non-saturated equilibrium wherein a tradeoff exists between the global network performance obtained at the equilibrium and the requested amount of signalling.

The Nash equilibrium is generally inefficient hence a pricing mechanism to obtain improvements in the users’ utilities with no pricing is proposed in [7] for flat-fading MACs using SUD. Along these lines and to further improve the overall performance of the network, a hierarchy is introduced between users. To this end, two schemes are proposed where a Stackelberg formulation with SUD at the receiver is first studied after which a more efficient receiver using successive interference cancelation (SIC) is investigated. It is worth noting that both approaches aim at improving the network equilibrium efficiency with the following features: a) a more thorough analysis of the uniqueness of the equilibrium is possible, b) energy-efficiency can be physically measured albeit pricing schemes, c) unlike pricing, only individual CSI is needed at each transmitter in the regime of non-saturated equilibria. This way, the intelligence is shared between the base station and the users.

- **Hierarchical game with SUD**

A stackelberg formulation of the power control game is proposed where one of the $K$ users is chosen to be the leader whereas the others are the followers. The receiver is not a player of the game at which SUD$^4$ is performed. The existence and uniqueness of the Stackelberg equilibrium is given by the following proposition:

**Proposition 1 (Existence and Uniqueness of an S.E) [6]** There is a unique Stackelberg equilibrium $p_{SE}^{SE} = (p_{i}^{SE}, p_{j}^{SE})$ in the proposed hierarchical game where user

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$^4$In SUD, the receiver has to remain neutral in the game and limit the receiver’s complexity as well as minimize the possible signalling costs.
i is the leader:

\[
p_{i}^{SE} = \frac{\sigma^2}{|h_i|^2} \frac{\gamma^* (1 + \beta^*)}{1 - (K - 1)\gamma^* - (K - 2)\beta^*}
\] (1.19)

and for each follower \( j \neq i \):

\[
p_{j}^{SE} = \frac{\sigma^2}{|h_j|^2} \frac{\beta^* (1 + \gamma^*)}{1 - (K - 1)\gamma^* - (K - 2)\beta^*}
\] (1.20)

if the following (sufficient) conditions hold:

\[
\frac{f''(0)}{f'(0)} \geq 2 \frac{(K-1)\beta^*}{1-(K-2)\beta^*}
\]

and

\[
\phi(x) = x \left[ 1 - \frac{(K-1)\beta^*}{1-(K-2)\beta^*} \right] f'(x) - f(x) \text{ has a single stationary point in } ]0, \gamma^*[,
\]

where \( \beta^* \) is the positive solution of the equation \( xf'(x) - f(x) = 0 \) and \( \gamma^* \) is the positive solution of the equation \( \phi(x) = 0 \).

Now, we turn to answer some important questions: 1) is it better to be a leader or follower in the proposed game? 2) What is the gain provided by the hierarchy? 3) Do all the players benefit from it? The answer to these questions is that it turns out that it is better to be a follower rather than a leader. This is given under the following proposition:

**Proposition 2 (Following is better than leading)** [6] Every user has always a better utility being chosen as a follower instead of a leader.

**Proof 1** \( u_{L}^{*SE} \) (resp. \( u_{F}^{*SE} \)) the utility of user \( i \in \{1, \ldots, K\} \) (resp. \( j \neq i \)) when he is chosen to be the leader (resp. a follower) of the game. First, we observe that at the Stackelberg equilibrium, the SINR of the leader and follower are:

\[
SINR_{L}^{*SE} = \gamma^* \quad \text{and} \quad SINR_{F}^{*SE} = \beta^*,
\] (1.21)

From [6], we have that:

\[
\forall x > 0 : x > \beta^* \leftrightarrow xf'(x) < f(x)
\] (1.22)

As for all \( x > 0, x \left[ 1 - \frac{(K-1)\beta^*}{1-(K-2)\beta^*} \right] f'(x) < xf'(x) \), from a single geometrical argument we see that \( \gamma^* < \beta^* \). This means that the SINR of the follower (i.e., \( \beta^* \)) is higher than the SINR of the leader (i.e., \( \gamma^* \)).
Additionally and interestingly enough, it is proven that both leaders and follower improve their utilities as compared to the non-cooperative approach. As a conclusion, the energy efficiency power allocation game yields better payoffs and lower transmit powers. This is rather unusual since for instance in duopoly [9] games only the leader benefits from hierarchy.

- **Hierarchical game with SIC**

In this approach, the BS implements SIC where users are ranked and decoded successively. For instance, in a 2 users case if user 1 is decoded first by the BS by treating user 2 as noise. Then, user 2 is decoded without interference. SIC is more complex and the decoding order has to be known to all users. But on the other hand SIC partially removes multi-user interference. Likewise, the existence and uniqueness of the N.E is given in the following proposition:

**Proposition 3 (Existence and Uniqueness of an N.E)** Let denote by \( i \) the index of the user decoded with rank \( K - i + 1 \) in the successive decoding procedure at the receiver. In the non-cooperative game with a SIC-based receiver where the utility is chosen to be given by Eq. (1.17) where the SINRs are those considered at the output of the SIC, there exists a unique (pure) NE \((p_1^{SIC}, ..., p_K^{SIC})\) given by:

\[
\forall i \in \{1, ..., K\}, \quad p_i^{SIC} = \frac{\sigma^2}{|h_i|^2} \beta^* \mu_i^{SIC} \tag{1.23}
\]

where \( \mu_i^{SIC} = (1 + \beta^*)^{i-1} \) is a penalty term due to multiple access interference (MAI).

Next, the authors give a comparison between SIC and SUD in which it turns out that every user prefers to be in the game with a receiver implementing SIC instead of SUD. This is due to the fact that in the SIC-based receiver, less transmit power is used and less interference is generated in the network. The authors also assess the influence of the degrees of freedom on the overall network energy-efficiency. To this end, they investigate both the social welfare and the energy-efficient of the equivalent virtual MIMO system. For the social welfare case, the best choice for a leader is found to be the one with lowest \( R_i|h_i|^2 \) and the best decoding order is to decode users in the increasing order of their energy weighted by the coding rate \( R_i|h_i|^2 \).
1.3.3 Hierarchical power allocation games in multi-user channels

In [5], Lai et al. focus on the problem of power allocation in the context of fading MACs. First, the problem is modeled as a static one-shot game [20] (without the intervention of the BS) where users selfishly compete between each other to maximize their respective payoffs using SUD subject to some power constraints given as follows:

$$\max_{p_i} \bar{R}_i(p_i, p_{-i}) \quad s.t. \quad p_i \in \mathcal{F}_i$$  \hspace{1cm} (1.24)

where $\mathcal{F}_i = \{p_i : \mathbb{E}_h(p_i) \leq P_i, p_i(h) \geq 0\}$ is the set of all feasible power control policies of user $i$, and $p_{-i}$ represents the power control policy of the other user. Finally, $\bar{R}_i = \mathbb{E}_h(R_i)$ is the average achievable rate with $h = [h_1, h_2]$.

In the 2-users case, the payoff of user $i$ is written as:

$$\bar{R}_i = \int \int \frac{1}{2} \log_2 \left( 1 + \frac{p_i(h_i, h_{-i})|h_i|^2}{\sigma^2 + p_{-i}(h_i, h_{-i})|h_{-i}|^2} \right) f(h_i, h_{-i}) dh_i dh_{-i}$$  \hspace{1cm} (1.25)

where $f(h_i, h_{-i})$ is the joint probability density function of the two fading coefficients. The solution to (1.25) is the well-known water-filling power allocation:

$$p_i(h_i, h_{-i}) = \left( \lambda_i - \frac{\sigma^2}{|h_i|^2} - \frac{p_{-i}(h_i, h_{-i})|h_{-i}|^2}{|h_i|^2} \right)^+$$  \hspace{1cm} (1.26)

where $x^+ = \max\{x, 0\}$ and $\lambda_i$ is the power level that satisfies the power constraint:

$$\int \int \left( \lambda_i - \frac{\sigma^2}{|h_i|^2} - \frac{p_{-i}(h_i, h_{-i})|h_{-i}|^2}{|h_i|^2} \right)^+ f(h_i, h_{-i}) dh_i dh_{-i} = \bar{P}_i$$  \hspace{1cm} (1.27)

Based on these equations, one can see that the optimal policy of each user depends largely on its guess of the other user’s policy. Therefore, each user will determine its policy and adjust its water-filling level to maximize its own average rate. Finally, at the N.E, the water-filling pair $(\lambda_i, \lambda_{-i})$ satisfies the two average power constraints with equality.

Lai et al. further demonstrate that the maximum sum-rate SP of the capacity region is
1.3. APPLICATION TO WIRELESS COMMUNICATIONS

the unique Nash equilibrium of the WF game. This result is rather remarkable because the selfish behavior of the users will lead them to jointly optimize the sum-rate of the channel. In other words, the user with strongest channel sees a relatively weak interference from the other user, and hence, decides to transmit with a higher power level. On the other hand, the other user sees a strong interference in addition to a weak channel, and hence decides to conserve the power for a later usage. Thus, the users reach a distributely opportunistic time-sharing equilibria.

In the second part of their work and motivated by the fact that in the one-shot approach, users treat each other as noise thereby unable to reach all points of the capacity region other than the sum-rate. Conversely, if the BS is introduced as an additionally player with successive cancelation decoding, all boundary points of capacity region can be achieved. This problem is modeled using a Stackelberg formulation where the BS is the game leader announcing the decoding order in the high-level game while the users are the followers in the low-level game.

- **Decoding order**: the base station divides the whole possible space of \((h_1, h_2)\) into two subsets \(D_1, D^c_1\). When \((h_1, h_2 \in D_1)\), the BS will decode user 1’s information first whereas \((h_1, h_2 \in D^c_1)\) implies decoding user 2’s signal first. Then, after the BS announces its strategy, \(D_1\), the multiple access users play the low-level using the Nash equilibrium concept.

- **Strategy**: the strategy space of user \(i\) is still \(F_i\) and the payoff function of user 1 is defined as the supremum of the achievable rate given by:

\[
\tilde{R}_1(D_1, p_1, p_2) = \int \int \frac{1}{2} \log_2 \left(1 + \frac{p_1(h_1, h_2)|h_1|^2}{\sigma^2 + p_2(h_1, h_2)|h_2|^2 I_{(h_1, h_2) \in D_1}}\right)f(h_1, h_2)dh_1dh_2
\]  

where \(I_{(h_1, h_2) \in D_1}\) is the indicator function. On the other hand, the payoff of the BS is written as:

\[
\mu_1 \tilde{R}_1(D_1, p_1, p_2) + \mu_2 \tilde{R}_2(D_1, p_1, p_2)
\]  

This payoff function has a natural economic interpretation as the revenue of the base
station where $\mu_i$ can be viewed as the payment that user $i$ owes per unit rate. The authors prove the existence of the N.E then the uniqueness of the N.E in the low-level game using the concept of admissible\footnote{Intuitively, this notion allows for eliminating Nash equilibria which are Pareto-dominated by other equilibrium points.} \cite{20} N.E. It is shown that the proposed Stackelberg game setup has a very desirable structure. For any given vector $\mu$, the existence of a BS policy which achieves a utility within an $\epsilon$-difference from the optimal one is guaranteed; and for every rational multiple access user, the optimal policy in the low-level game is unique. Therefore, the users will have no difficulty in deciding the power and rate levels in a distributed way. As a result, the introduction of the BS as a game leader enlarges the achievable rate region as compared with the Nash game. Nevertheless, it is shown that not all the points of capacity regions are achieved, hence the game is reformulated as a \textit{dynamic and repeated game} \cite{20}.

In the last part of their contribution, Lai et al. extend the work to $N_r$ antennas at the BS. In the \textit{one-shot} game, the authors prove the uniqueness of the N.E (i.e., the one achieving the sum-rate point SP). However, the achievable rates are strictly smaller than the rates corresponding to SP. Moreover, and in contrast to the single antennas case in which the power allocation strategy was time-sharing, there are $\min(N, N_r)$ degrees of freedom, and hence more than one user is allowed to transmit at any fading state. In the Stackelberg game, a unique admissible Nash equilibrium for the low-level game is proven. It also turns out that the Stackelberg game achieves the two corner points of the capacity region but does not achieve the maximum sum-rate point. Additionally, unlike the static game, the users can achieve any point on the capacity region as a subgame perfect equilibrium using the same strategies as in the single antenna case. Finally, we note that the more generalized MIMO case has been recently treated in \cite{32}.

On the other hand, the problem of spectrum sharing for the frequency-selective IFC has been well studied in \cite{21}, \cite{22}, \cite{33}-\cite{35} among others, as well as the flat-fading case in \cite{16}. Therein, the spectrum sharing problem is modeled as a strategic non-cooperative (\textit{one-shot}) game\footnote{See \cite{4}, for a good survey on the application of game theory for dynamic spectrum sharing.} in which transmitters selfishly maximize their utility function subject to the power
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Constraint \( \bar{P}_i, i \in \{1, \ldots, K\} \):

\[
\text{maximize } R_i = \max_{p^n_i} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_{n,i}^n|^{2}p^n_i}{\sigma^2_n + \sum_{j \neq i} |h_{n,j}^n|^{2}p^n_j} \right) \\
\text{subject to } \sum_{n=1}^{N} p^n_i \leq \bar{P}_i \\
p^n_i \geq 0 \tag{1.30}
\]

The solutions to (1.30) are given by the water-filling power allocation:

\[
p^n_i = \left( \frac{1}{\mu_i} - \frac{\sigma^2_n + \sum_{i} |h_{n,i}^n|^{2}p^n_{-i}}{|h_{n,i}^n|^{2}} \right)^+ \quad i = 1, \ldots, K \quad n = 1, \ldots, N \tag{1.31}
\]

where \((x)^+ = \max\{x, 0\}\) and \(\mu_i > 0\) is the Lagrangian multiplier chosen to satisfy the power constraint: \(\sum_{n=1}^{N} p^n_i = \bar{P}_i\).

It turns out that under certain channel realizations, the N.E of the game is unique whereas other channel conditions exhibit several Nash equilibria. This renders the game unpredictable since players are distributed and there is no central entity enforcing a certain equilibrium.

In the case of arbitrary number of transmitters \(K\), the results for the 2 transmitters/operators and 2 carriers case carry over where the sufficient conditions for the uniqueness are given\(^7\) by:

\[
\sum_{i=1, j \neq i}^{K} \frac{|h_{n,j}^n|^{2}}{|h_{n,i}^n|^{2}} < 1, \quad n = 1, \ldots, N \tag{1.32}
\]

The physical meaning of (1.32) is that the uniqueness of the N.E is ensured if the links are sufficiently far from each other.

A Stackelberg game \(\Gamma^{SG} \triangleq [\mathcal{K}, \{P_i\}_{i \in \mathcal{K}}, \{U_i\}_{i \in \mathcal{K}}]\) is proposed to model the spectrum sharing problem where the primary operator 1 is the leader and secondary operator 2 is the follower. The Stackelberg spectrum sharing game is formulated as follows. First, in the high-level problem (1.33), primary operator 1 maximizes his own utility function and in the

\(^7\)In \([33]\), other sufficient conditions are given therein.
low-level problem (1.34) secondary operator 2 maximizes his own utility taking into account the optimal power allocation of operator 1, $p_1^{SE}$. By denoting $(p_1^{SE}, p_2^{SE})$ as the Stackelberg Equilibrium, the rate optimization problem for operator 1 (leader) is written as:

$$\max_{p_1^n} \sum_{n=1}^N \log_2 \left( 1 + \frac{|h_{n1}|^2 p_1^n}{\sigma_n^2 + |h_{n21}|^2 p_2^n (p_1^{SE})} \right)$$

$$\sum_{n=1}^N p_1^n \leq \bar{P}_1$$

$$p_1^n \geq 0$$

The rate optimization problem for operator 2 is written as:

$$\max_{p_2^n} \sum_{n=1}^N \log_2 \left( 1 + \frac{|h_{n2}|^2 p_2^n}{\sigma_n^2 + |h_{n12}|^2 (p_1^{SE})} \right)$$

$$\sum_{n=1}^N p_2^n \leq \bar{P}_2$$

$$p_2^n \geq 0$$

where $p_2^{SE} = BR_2(p_1^{SE})$.

Using backward induction and given the best response of operator 2, (1.33) can be rewritten as:

$$\max_{p_1^n} \sum_{n=1}^N \log_2 \left( 1 + \frac{|h_{n1}|^2 p_1^n}{\sigma_n^2 + |h_{n21}|^2 (p_1^{SE})} \right)$$

$$\sum_{n=1}^N p_1^n \leq \bar{P}_1$$

$$p_1^n \geq 0$$

The Stackelberg sharing game therefore boils down to solving (1.35) where several cases are considered the details of which are given in [21].

The inter-operator spectrum sharing in the context of two operators can be extended to the general case with arbitrary $K$ operators sharing the spectrum. The problem is formulated
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Table 1.1: Algorithm 1

<table>
<thead>
<tr>
<th>Algorithm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>initialize $\lambda, P_1, P_2$</td>
</tr>
<tr>
<td>repeat</td>
</tr>
<tr>
<td>for $n = 1..N$</td>
</tr>
<tr>
<td>set $p_1^n = \text{arg max}<em>{p_1^n} \sum</em>{n=1}^{N} \log_2 \left(1 + \frac{</td>
</tr>
<tr>
<td>by keeping $p_1^1, .., p_1^{n-1}, p_1^{n+1}, p_1^N$ fixed.</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>until $(p_1^1, .., p_1^N)$ converges</td>
</tr>
</tbody>
</table>

in the same way where the leader’s optimization problem is written as:

$$\max_{p_1^n} \sum_{n=1}^{N} \log_2 \left(1 + \frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2 + \sum_{j \neq 1} |h_{j1}^n|^2 p_j^n(p_j^n)}\right)$$

$$\sum_{n=1}^{N} p_1^n \leq \bar{P}_1$$

$$p_1^n \geq 0$$

and $p_{j}^{SE} = BR_j(p_1^{SE}, .., p_{-j}^{SE})$ is a function of $p_1^n$.

Solving (1.36) becomes much more involved in the general case in which the utility function of the primary operator is non-convex ($p_1^n$ is function of $p_1^n$). Nevertheless, sub-optimal and low-complexity methods provide neat solutions to solve the problem based on lagrangian duality [30].

The lagrangian of (1.36) is given by:

$$g(\lambda) = \max_{p_1^n} \mathcal{L}(p_1^n, \lambda)$$

$$= \max_{p_1^n} \sum_{n=1}^{N} \log_2 \left(1 + \frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2 + \sum_{j \neq 1} |h_{j1}^n|^2 p_j^n(p_j^n)}\right) + \lambda \left(\bar{P}_1 - \sum_{n=1}^{N} p_1^n\right)$$

where $\lambda$ is the lagrangian dual variable associated with the power constraint.

Consequently, solving the Stackelberg problem is done by locally optimizing the lagrangian function (1.37) via coordinate descent [23]. For each fixed set of $\lambda$, we find the optimal $p_1^1$ while keeping $p_1^2, .., p_1^N$ fixed, then find the optimal $p_1^2$ keeping the other $p_1^n (n \neq 2)$ fixed.
and so on. Such process is guaranteed to converge because each iteration strictly increases the objective function. Finally, \( \lambda \) is found using sub-gradient [30] method.

For sake of illustration, Figure 1.1 depicts the best and worst N.E where the best N.E refers to the equilibrium maximizing the sum-rate of both operators whereas the worst N.E case minimizes it. It is also worth noting that the worst Nash equilibrium acts like a lower-bound for the Nash equilibrium. Furthermore, the Stackelberg approach is closer to the centralized approach as compared to the selfish case. This is due to the fact that in the Stackelberg approach, operators take into account other operators’ strategies whereas in the selfish case, operators behave carelessly by using water-filling. On the other hand, Figure 1.2 depicts the cumulative distribution function (cdf) of the ratio between the achievable rates of the hierarchical and non-cooperative approaches. In this scenario, we assume \( K = 4 \) operators with 1 primary operator and 3 secondary wireless operators sharing the same spectrum composed of \( N = 5 \) carriers. As can be seen, the primary operator (operator 1) always improves his achievable rate compared to the selfish approach.

### 1.3.4 Base station location games and hierarchy

In [44], the authors study another application of hierarchy being the impact of decision making arising in the uplinks of cellular networks in which hierarchy is accounted for. Firstly, the problem of SINR association is addressed where given multiple BSs capable of providing services to a mobile, to which BS should a mobile connect? This problem is studied in a non-cooperative context where each mobile connects to the BS that provides it with the best SINR (PA game). The associations as a result determine the cells corresponding to each BS. In their study, two interference models are assumed wherein mobiles that connect to a particular BS may or may not cause interference to the other BS depending on whether the BSs operate on the same (single-frequency case) or different frequency bands (two-frequencies case).

In the single frequency case, the power from all the mobiles is received at both BSs where the total received power at BS \( j \) located at \( x_j \) on the segment \([−L, L] \) is \( E(x_j, [−L, L]) = \)
In contrast, the total interference at each BS in the two-frequencies case depends on the association decision of mobiles. The interference power at BS $j$ is the total power received at the BS from all mobiles that actually associate with it and the total received power at BS $j$ is $E(x_j, A_j)$ where $A_j \subset [-L, L]$ is such that the mobiles in $A_j$ are associated with BS $j$ ($A_j$ is also referred to as cell $j$).

Let $I_j$ be the set of interferers at BS $j$. If the mobiles at point $y$ are associated with BS $j$, the SINR density is given by:

$$\text{SINR}(y, x_j, I_j) = \frac{g(y - x_j)}{E(x_j, I_j) + \sigma^2}$$ (1.38)

A mobile at $y \in [-L, L]$ will therefore prefer to associate with BS 1 rather than BS 2 if $\text{SINR}(y, x_1, I_1) > \text{SINR}(y, x_2, I_2)$. In the single frequency case $I_j = [-L, L]$, thus the SINR density at a location at BS $j$ is fixed, whereas in the two-frequencies case, $I_j = A_j, j = 1, 2$. Hence, the SINR density at a location seen at BS $j$ is a function of the cell $A_j$. Furthermore, the cell partition $(A_1, A_2)$ is said to be an SINR-equilibrium if the equilibrium holds: $y \in A_1$ if and only if $\text{SINR}(y, x_1, I_1) \geq \text{SINR}(y, x_2, I_2)$.

The authors also addressed the problem of BS placements taking into account the SINR-equilibrium when mobiles associate to maximize their SINR density. In this context, the BSs play a location game in which BS $j$ decides to place itself at $(x_j, 1)$ where $x_j \in \mathbb{R}, j = 1, 2$. The utility function of the BS is a monotone function of the aggregate throughput of all the mobiles associated with it. For BS $j$ with cell $A_j$ and interferers $I_j$, the utility is defined as:

$$\frac{1}{2} \int_{A_j} \text{SINR}(y, x_j, I_j) dy = \frac{1}{2} \int_{A_j} \frac{g(y - x_j) dy}{E(x_j, I_j) + \sigma^2}$$ (1.39)

Once the BSs choose their locations $A_j$ and $I_j$, the utility function of BS $j$ is determined by the association game played by the mobiles. This is formulated as a Stackelberg game where the two BSs are the leaders and the mobiles are the followers.

Numerical results conducted for the SINR association game in the single-frequency case show rather interesting findings. 1) If a mobile is very close to a BS, the path gain $MS \rightarrow BS$ will be very high. So, the mobile connects to this BS despite the fact that the interference
suffered by this BS is relatively high. 2) If a mobile is located sufficiently far from both BSs, the relative difference in the powers received at the BSs will be small. Thus, the mobile will prefer to connect to the BS that suffers from less interference. 3) If a mobile is at moderate distance from both BSs, it takes into account both path gains to the BSs and interference suffered at the BSs to make an association decision. Finally, the authors also provide closed form expressions for cell boundaries.

In the hierarchical game with one BS, the utility function of the BS is written as:

\[
\frac{1}{2} \int_{-L}^{L} g(y - x_j) dy = \frac{1}{2} \frac{E_0(x)}{E_0(x) + \sigma^2}
\]

which is maximized when \( E_0(x) \) is maximized, i.e., at \( x = 0 \). Despite the high interference, the origin is the best location to maximize the utility given the nature of the utility function.

In contrast and in the case of two cooperating BSs, the optimal joint placement of the BSs to maximize the sum-utility is therein studied. It turns out that the sum utility is maximized when the BSs are equidistant from the vertical axis and the SINR equilibrium cells are \([-L, 0]\) and \([0, L]\) in the symmetric case. However, in the case when the two BSs are non-cooperative moving simultaneously picking their respective locations accounting for the SINR-equilibrium associations of mobiles, the utility of BS 2 is quite robust to placement errors around the best response location, for the indicated values of BS 1 locations.

Similarly, the authors investigate the second case when the BSs operate in disjoint frequency bands. First, the SINR equilibrium association is numerically computed. With hierarchy, the authors give the optimal location for both cooperative BSs so as to maximize their sum utility whereas when the BSs are non-cooperative numerical results show the existence of a unique equilibrium.

In the last part of their work, the authors investigate the effect of employing SIC decoding by the BSs. A mobile first associates with a BS then the BSs choose an arbitrary decoding order. This can be interpreted by the fact that the mobile opportunistically believes that it will be decoded last and therefore expects to see an SINR density of \( g(y - x_j) / \sigma^2 \) with BS \( j \). The optimal cooperative locations are characterized as well as all pure-strategy competitive
1.4 Concluding remarks and open issues

In this chapter, we have looked at hierarchical power allocation games in which the concept of hierarchy is introduced. Game theory is a natural paradigm to study these network interactions in which nodes/terminals/transmitters compete with each other for the same resources. The existence and uniqueness of the Nash and Stackelberg equilibria is a nice feature as it predicts the outcome of the game and gives insights on its convergence.

We conclude this chapter by highlighting few open issues related to hierarchical power allocation problems. It has been shown in various applications that hierarchy is a relevant aspect in wireless communication. Since more and more terminals are foreseen to be deployed in unlicensed bands, more advanced sharing mechanisms have to be investigated to avoid the tragedy of the commons [18] due to the fact that device designers lack an incentive to conserve the shared spectrum resource. Other potential area of interest have to be sought after. For instance, due to the increasing interest for femtocells [46] in which the macro-user is given a higher priority than the femto-user, the Stackelberg framework can readily be applied to solve the radio resource allocation problem. This will rely on signalling issues.

References

Chapter 1. Hierarchical Power Allocation Games


on Selected Areas of Comm., vol. 25, no. 3, pp. 517-528, April 2007.


[44] E. Altman, A. Kumar, C. Singh and R. Sundaresan, ”Spatial SINR Games Combining
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Figure 1.1: Average achievable rate for both users versus the signal-to-noise ratio for the centralized and Stackelberg approach. Moreover, the best and worst Nash equilibria for the non-cooperative game are illustrated.

Base Station Placement and Mobile Association," IEEE Infocom, Rio de Janeiro, Brazil, April 19-25, 2009.


Figure 1.2: Cumulative distribution function (CDF) of the ratio of the rates between the hierarchical (Stackelberg) and non-cooperative (selfish) approaches.