Global Asymptotic Stabilization of a PVTOL Aircraft Model with Delay in the Input

Rogelio Francisco[1], Frédéric Mazenc[2], and Sabine Mondié[1]

[1] Departamento de Control Automático CINVESTAV-IPN, Av. IPN 2508, A.P. 14-740, 07300 México, D.F., México
rfrancisco@ctrl.cinvestav.mx, smondie@ctrl.cinvestav.mx
[2] Projet MERE INRIA-INRA, UMR Analyse des Systèmes et Biométrie, INRA, 2, pl. Viala 34060 Montpellier, France
mazenc@helios.ensam.inra.fr

Summary. A model of PVTOL aircraft with two delayed inputs is considered. The origin of this system is globally asymptotically and locally exponentially stabilized by bounded control laws. The explicit expressions of the control laws applied are determined through recent extensions of the forwarding approach to systems with a delay in the input. In a second step, the output feedback stabilization problem consisting in globally asymptotically stabilizing the origin of the PVTOL when the variables of velocity are unmeasured is solved through a recent technique which extensively exploits the presence of positive delays in the inputs.

Keywords: stabilization, PVTOL aircraft model, nonlinear feedforward system, output feedback.

1 Introduction

The feedforward systems are nonlinear systems described by equations having a specific triangular structure which, in general, cannot be linearized. The problem of the global asymptotic stabilization by state feedback of these triangular equations in the absence of delay has been studied by many researchers [18, 8, 16, 5, 19], during the last decade. The techniques of stabilization of feedforward systems have been successfully applied to different physical devices such as, for example, ‘ the card-pendulum system ’(see [9]), ‘ the Ball and beam ’ with a friction term (see [16]), ’ the TORA system ’ (see [16]) and ’ the PVTOL ’ (*Planar Vertical Takeoff and Landing Aircraft*), (see [18]).

Three recent works [10, 12, 14] are devoted to the problem of designing globally asymptotically stabilizing control laws for particular families of feedforward systems with an arbitrarily large delay in the input: this problem is solved for chains of integrators in [10, 14] and for nonlinear feedforward systems admitting a chain of integrators as linear approximation at the origin in [12, 13]. The basic idea of these three papers consists in selecting, according to the value of the delay, appropriate stabilizing control laws in a family of control laws whose explicit formulae generalize those of the control laws provided by A. Teel in [17].

J. Chiasson and J.J. Loiseau (Eds.): Appl. of Time Delay Systems, LNCIS 352, pp. 343–356, 2007.
springerlink.com

In the present work we will use the aforementioned theoretical results, and especially the one of [13], to stabilize a PVTOL model, when the control inputs are subject to delays. The PVTOL aircraft model is well-known by the control community. Due to the fact that flight control is an essential control problem, this simple model, which retains main features that must be considered when designing control laws for a real aircraft, has been studied extensively by many researchers. Some of the works devoted to this system are the following. In 1992 J. Hauser et al. [2] developed for this model an approximate input-output linearization procedure which results in bounded tracking and asymptotic stability. In 1996 A. Teel [18] illustrated his nonlinear small gain theorem based approach for the stabilization of feedforward systems by applying it to the PVTOL aircraft. In 1996 P. Martin et al. [7] proposed an extension of [2] relying extensively on the concept of flatness. In 1999, F. Lin et al. [4] studied the robust hovering control of the PVTOL and designed a nonlinear state feedback by applying an optimal control approach. The recent publications by L. Marconi et al. [6] within an internal model approach and by K.D. Do et al. [1], who have solved an output feedback tracking problem, show that this system still captures the attention of researchers.

Observe that, due to the number of papers devoted to the PVTOL system, the list of works on the PVTOL aircraft we give is not exhaustive. However, to the best of our knowledge, all the theoretical results available in the literature on the asymptotic stabilization of the PVTOL assume that there is no delay in the inputs. Nevertheless, such a delay, due to sensors and information processing, is often present in practice. This is in particular the case of the experimental PVTOL setup presented in the work of Palomino et al. [15] where the position and roll angle of the system are measured with the help of a vision system that induces a delay of approximately $40ms$.

The main features of our contribution can be summarized as follows. In the first part of the work, we construct state feedbacks which globally asymptotically and locally exponentially stabilize the origin of the equations modelling the PVTOL when there are known delays in the inputs. These constructions extensively rely on the control design techniques proposed in [12], [13], [10]. The control laws obtained that way are bounded and involve a distributed term. Moreover they depend on the variables of position and velocity. In the second part of the work, we complement this result by showing that using the presence of known non-zero delays in the inputs (or by introducing artificially delays in the inputs), one can determine globally asymptotically and locally exponentially stabilizing control laws depending only on the variables of position and not on the variables of velocity, which in practice cannot be easily measured. This result is proved through ideas borrowed from the recent works [11] and [3] on the output feedback stabilization of linear systems by means of delayed feedbacks. The main feature of the original approach proposed in these works is that it does not rely on the construction of an observer or on the introduction of dynamic extensions but only on the presence of a delay. In the present paper, it is applied for the first time to a nonlinear system. This strategy of output feedback stabilization for the PVTOL has clearly no similarity with the one adopted in [1], since the latter relies on the construction of an observer.

The paper is organized as follows. In Section 2, we recall the main theoretical result which is used to construct the control laws. The simplified PVTOL aircraft model that is analyzed in this work is presented in Section 3. The control laws and the state reconstructor for the PVTOL aircraft model are designed respectively in Sections 4 and 5. Simulation results are presented in Section 6. The paper ends with some concluding remarks in Section 7.

Technical Preliminaries

1. A function $\gamma(X)$ is of order one (resp. two) at the origin if for some $c > 0$, the inequality $|\gamma(X)| \leq c|X|$ (resp. $|\gamma(X)| \leq c|X|^2$) is satisfied on a neighborhood of the origin.
2. The argument of the functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $f(x(t))$ by simply $f(t)$ or $f(\cdot)$.
3. By $\sigma : \mathbb{R} \to \mathbb{R}$ we denote a saturation function which satisfies
a) $\sigma(\cdot)$ is odd, nondecreasing and of class C^1,
b) $0 \leq \sigma'(s) \leq 1$, $\forall s \in \mathbb{R}$,
c) $\sigma(s) = 1$ for all $s \geq \frac{21}{20}$ and $\sigma(s) = s$ for all $s \in \left[0, \frac{19}{20}\right]$.
4. By $\sigma_i : \mathbb{R} \to \mathbb{R}$ we denote the functions

$$\sigma_i(s) := \varepsilon_i \sigma\left(\frac{1}{\varepsilon_i} s\right), \; \varepsilon_i = \frac{1}{20^{n-i+1}}, \; i = 1, \ldots, n. \tag{1}$$

2 Theoretical Results

In this section, we recall the main stabilization result of [13] for nonlinear feedforward systems with a delay in the input and subject to vanishing perturbations. It is a generalization of the result presented in [12] for nonlinear feedforward systems in absence of vanishing perturbations, which in turn is a generalization of the recursive methodology developed in [11] to solve the problem of stabilizing chains of integrators.

Theorem 1. *[13] Consider the following feedforward system*

$$\begin{cases} \dot{x}_1(t) &= x_2(t) + h_1(x_2(t), \ldots, x_n(t)) + r_1(t), \\ \dot{x}_2(t) &= x_3(t) + h_2(x_3(t), \ldots, x_n(t)) + r_2(t), \\ \vdots & \\ \dot{x}_{n-1}(t) &= x_n(t) + h_{n-1}(x_n(t)) + r_{n-1}(t), \\ \dot{x}_n(t) &= u(t-\tau), \end{cases} \tag{2}$$

where $x_i \in \mathbb{R}$, $u \in \mathbb{R}$ is the input, $\tau \geq 0$ is the delay and where each function $h_i(\cdot)$ is a function of a class C^2 and of order 2 at the origin, that satisfies the inequality

$$|h_i(x_{i+1}, x_{i+2}, \ldots, x_n)| \leq M(x_{i+1}^2 + x_{i+2}^2 + \cdots + x_n^2) \tag{3}$$

where M is a strictly positive constant when $|x_j| \leq 1, \quad j = i+1, \ldots, n$, and where each function $r_i(\cdot)$ is a function continuously differentiable and such that, for some real-valued nonnegative and nonincreasing function $R \in L^2[0, +\infty)$ the inequalities

$$|r_i(t)| \leq R(t) \tag{4}$$

are satisfied for all $t \geq 0$. Consider the control law bounded in norm

$$\begin{aligned} u(x_1, x_2, \ldots, x_n) = -\tfrac{L}{Mk^n}\sigma_n(p_n(k^{n-1}\tfrac{M}{L}x_n) + \ldots \\ +\sigma_{n-1}(p_{n-1}(k^{n-2}\tfrac{M}{L}x_{n-1}, k^{n-1}\tfrac{M}{L}x_n) + \ldots \\ +\sigma_1(p_1(\tfrac{M}{L}x_1, \ldots, k^{n-2}\tfrac{M}{L}x_{n-1}, k^{n-1}\tfrac{M}{L}x_n))) \ldots) \end{aligned} \tag{5}$$

where

$$p_i(x_i, ..., x_n) = \sum_{j=i}^{n} \frac{(n-i)!}{(n-j)!(j-i)!} x_j,$$

where the functions $\sigma_i(\cdot)$ are the functions defined in the preliminaries (see (1)) and

$$k \geq \frac{\tau}{\min\left\{\frac{1}{16n^3\left[4n\sqrt{n}(1+n^2)^{n-1}+1\right]^2}, \frac{1}{4.20^{n+1}n(n+2)}\right\}}, \tag{6}$$

$$0 < L \leq \min\left\{\frac{\eta k}{n^3(n!)^3}, \frac{Mk}{(n+1)!}, M\right\}, \tag{7}$$

$$0 \leq \eta \leq \min\left\{\frac{1}{8(1+n^2)^{n-1}}, \frac{1}{10.20^n n}\right\}.$$

Then all the trajectories of the system (2) in closed-loop with the control law (5) converge to the origin. Moreover, the origin of the system (2) in closed-loop with the control law (5) is globally uniformly asymptotically and locally exponentially stable when each function $r_i(\cdot)$ is identically equal to zero.

3 The PVTOL Model and Problem Statement

Aircraft control is a challenging field of control theory. Given the complexity of the systems describing the behavior of aircraft, it is convenient to study simplified models of them that contemplate a specific number of state variables and controls which capture the essential features of the systems for control purposes. The simplified model of PVTOL we consider in this work is the following

$$\left.\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= u_1(t-\tau_1)\sin\theta, \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= u_1(t-\tau_1)\cos\theta - 1, \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= u_2(t-\tau_2). \end{aligned}\right\} \tag{10}$$

The variables x_1, y_1 denote the horizontal and the vertical positions, θ is the roll angle that the aircraft makes with the horizon, u_1, u_2 are the control inputs and $\tau_2 > 0, \tau_1 > 0$ are the delays. The control input u_1 is the thrust (directed out of the bottom of the aircraft) and u_2 is the angular acceleration (rolling moment).

In the next two sections, we will address the following problems:

Problem 1: *Construct state feedbacks which globally uniformly asymptotically and locally exponentially stabilizes the system (8), (9), (10) when there are delays in the inputs.*

Problem 2: *Construct output feedbacks which globally uniformly asymptotically and locally exponentially stabilize the system (8), (9), (10) with θ, y_1, x_1 as output variables when there are delays in the inputs.*

4 Stabilization Result

This section is devoted to Problem 1. Through the result which has been recalled in section 2, we will establish the following result.

Theorem 2. *Consider the system (8), (9), (10) with the delays $\tau_1 = 0.2, \tau_2 = 0.3$. The origin of this system in closed-loop with the control laws*

$$u_1 = u_{1s}(y_2(t-\tau_1), y_1(t-\tau_1), \hat{\theta}(t-\tau_2)), \tag{11}$$

$$u_2 = u_{2s}(\omega(t-\tau_2), \theta(t-\tau_2), x_2(t-\tau_2), x_1(t-\tau_2)) \tag{12}$$

with

$$\begin{aligned} u_{2s}(\omega, \theta, x_2, x_1) = & -\tfrac{L}{Mk^4}\sigma_4(k^3\tfrac{M}{L}\omega + \sigma_3(k^3\tfrac{M}{L}\omega \\ & +k^2\tfrac{M}{L}\theta + \sigma_2(k^3\tfrac{M}{L}\omega + 2k^2\tfrac{M}{L}\theta + k\tfrac{M}{L}x_2 \\ & +\sigma_1(k^3\tfrac{M}{L}\omega + 3k^2\tfrac{M}{L}\theta + 3k\tfrac{M}{L}x_2 + \tfrac{M}{L}x_1)))), \end{aligned} \tag{13}$$

with $M = 0.6$ and $L = 6.45 \times 10^{-10}$, $k = 7.5931 \times 10^{12}$ and

$$u_{1s}(y_2, y_1, \hat{\theta}) = \frac{1 + v_{1s}(y_2, y_1)}{\cos(\sigma(\hat{\theta}))}, \tag{14}$$

with

$$v_{1s}(y_2, y_1) = -\sigma_2(y_2 + \sigma_1(y_2 + y_1)) \tag{15}$$

and

$$\begin{aligned} \hat{\theta}(t-\tau_2) = & \,\theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\ & -\textstyle\int_{t-\tau_2}^{t}(s-t)u_{2s}(s-\tau_2)ds \end{aligned} \tag{16}$$

is globally uniformly asymptotically and locally exponentially stable.

Remark

1. For the sake of simplicity, we have restricted our attention to the case where $\tau_1 = 0.2, \tau_2 = 0.3$. However, one can easily deduce from the proof of Theorem 2 that for any values of τ_1, τ_2, Problem 1 can be solved.

2. From a practical point of view, the smallness of the size of the control law $u_{2s}(\cdot)$ is a drawback. It is important to observe that this drawback can be overcome. Indeed, by constructing a control law by means of the key ideas of Theorem 1 but by taking advantage of the specificity of the nonlinearities of the system (21), one can obtain a control law $u_{2s}(\cdot)$ with respectively much larger and much smaller values for the parameters L and k. For the sake of simplicity, we do not have performed this simple but lengthy construction of feedback and have instead directly applied Theorem 1.

Proof. The proof splits up into three steps. In Step 1 and Step 2, we establish that the control law defined in (12) ensures that the solutions of the subsystem (10) enter in finite time a particular neighborhood of the origin. Next, we show that this property implies that the control law defined in (11) stabilizes the subsystem (9). Then the problem considered reduces to the stability analysis of a four dimensional feedforward system. This analysis is carried out in Step 3.

Step 1

In Appendix A, we establish the following result.

Lemma 1. *The control law defined in (11) is well defined. The trajectories of system (8), (9), (10) in closed-loop with the bounded feedbacks (11), (12) are defined for all $t \geq 0$. Moreover, there exists $T \geq 2\tau_2$ such that*

$$|\theta(t)| \leq \frac{\pi}{4}\,, \ \forall t \geq T. \tag{17}$$

Step 2

One can establish that, for all $t \geq 2\tau_2$,

$$\hat{\theta}(t - \tau_2) = \theta(t), \tag{18}$$

by observing that, when $t \geq 2\tau_2$,

$$\begin{aligned}
\theta(t) &= \theta(t-\tau_2) + \textstyle\int_{t-\tau_2}^{t} \dot{\theta}(s)ds \\
&= \theta(t-\tau_2) + \textstyle\int_{t-\tau_2}^{t} \omega(s)ds \\
&= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\
&\quad + \textstyle\int_{t-\tau_2}^{t} [\omega(s) - \omega(t-\tau_2)]ds \\
&= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\
&\quad + \textstyle\int_{t-\tau_2}^{t} \left(\int_{s}^{t-\tau_2} \dot{\omega}(l)dl\right) \\
&= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\
&\quad - \textstyle\int_{t-\tau_2}^{t} \left(\int_{t-\tau_2}^{s} u_2(l-\tau_2)dl\right) \\
&= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\
&\quad - \textstyle\int_{t-\tau_2}^{t} (s-t)u_{2s}(s-\tau_2)ds.
\end{aligned}$$

Equality (18), the definition of $\sigma(\cdot)$ and Lemma 1 ensure that for all $t \geq T$,

$$u_{1s}(t-\tau_1) = \frac{1 + v_{1s}(t-\tau_1)}{\cos(\theta(t))} \tag{19}$$

which implies that for all $t \geq T$, the system (9) simplifies as

$$\begin{aligned}\dot{y}_1 &= y_2(t),\\ \dot{y}_2 &= v_1(t-\tau_1)\\ &= -\sigma_2(y_2(t-\tau_1)+\sigma_1(y_2(t-\tau_1)+y_1(t-\tau_1))).\end{aligned} \tag{20}$$

Using Theorem 1 (or the main result of [10]), one can prove that this system is globally uniformly asymptotically and locally exponentially stable.

Step 3

According to (19), the system (8), (10) in closed-loop with (11), for all $t \geq T + 2\tau_2$, is described by the equations

$$\begin{cases}\dot{x}_1 &= x_2(t),\\ \dot{x}_2 &= (1+v_{1s}(t-\tau_1))\tan\theta(t),\\ \dot{\theta} &= \omega(t),\\ \dot{\omega} &= u_{2s}(t-\tau_2),\end{cases}$$

or, equivalently, by the equations

$$\begin{cases}\dot{x}_1 &= x_2(t),\\ \dot{x}_2 &= \theta(t) + (\tan\theta(t) - \theta(t)) + v_{1s}(t-\tau_1)\tan\theta(t),\\ \dot{\theta} &= \omega(t),\\ \dot{\omega} &= u_{2s}(t-\tau_2).\end{cases} \tag{21}$$

Observe that the inequalities

$$|\tan\theta - \theta| \leq \int_0^{|\theta|} \tan^2(l)dl \leq 0.6\theta^2$$

hold for all $\theta \in [-1,1]$. It follows that the function $\tan\theta - \theta$ is of class C^2 of order 2 at the origin: it satisfies the requirement (3) imposed on the function $h_2(\cdot)$ in Theorem 1. Notice also that in (21), the functions corresponding to $h_1(\cdot)$ and $h_3(\cdot)$ in Theorem 1 are identically equal to zero. Moreover, we know that the real-valued functions $y_1(t)$, $y_2(t)$ converge exponentially to zero and that for all $t \geq T$, $|\tan\theta(t)| \leq 1$. Therefore $v_{1s}(t-\tau_1)\tan\theta(t)$ converges exponentially to zero: it follows that this function belongs to $L^2\,[0,+\infty)$ and thereby can be regarded as a bounded vanishing disturbance ($r_2(t)$ in Theorem 1). Then, using Theorem 1, one can check that all the trajectories of the feedforward system (21) converge to the origin and besides that the system (8), (9), (10) in closed-loop with the feedbacks (12), (15) is globally uniformly asymptotically and locally exponentially stable. (The value of the constant M, in the particular case of the system (21), is $M = 0.6$.) This concludes the proof.

5 A State Reconstructor for the PVTOL

This section is devoted to Problem 2 (see the end of Section 3). We show that one can solve the problem of stabilizing the PVTOL system when only the variables of position are available by measurement. The approach consists in evaluating the exact values of the variables of velocity through a state reconstructor for each subsystem (8), (9), (10). We show that, when the delays are known, the knowledge of the positions and roll angle of the aircraft which correspond to the states $x_1(t)$, $y_1(t), \theta(t)$ along with the control inputs $u_1(t)$ and $u_2(t)$ at present and past time instants is sufficient to determine the derivatives $x_2(t), y_2(t), \omega(t)$. The approach draws inspiration from the ideas on output feedback stabilization used in [11] for the case of a bounded input delayed simple oscillator and in [3] for the case of multiple oscillators and chains of integrators.

Theorem 3. *Consider the system (8), (9), (10) with delays* $\tau_1 = 0.2, \tau_2 = 0.3$. *The origin of this system in closed-loop with the control laws*

$$u_2(t-\tau_2) = u_{2s}(\overline{\omega}(t-\tau_2), \theta(t-\tau_2), \overline{x}_2(t-\tau_2), x_1(t-\tau_2)), \tag{22}$$

$$u_1(t-\tau_1) = u_{1s}(\overline{y}_2(t-\tau_1), y_1(t-\tau_1), \overline{\hat{\theta}}(t-\tau_2)), \tag{23}$$

with

$$\begin{aligned} \overline{x}_2(t) &= \tfrac{1}{\tau_1}\Big[x_1(t) - x_1(t-\tau_1) - \\ &\quad \textstyle\int_{t-\tau_1}^{t} \left(\int_t^s u_{1s}(l-\tau_1)\sin\theta(l)dl\right) ds\Big], \\ \overline{y}_2(t) &= \tfrac{1}{\tau_1}\Big[y_1(t) - y_1(t-\tau_1) - \\ &\quad \textstyle\int_{t-\tau_1}^{t} \left(\int_t^s (u_{1s}(l-\tau_1)\cos\theta(l) - 1)dl\right) ds\Big], \\ \overline{\omega}(t) &= \tfrac{1}{\tau_2}\Big[\theta(t) - \theta(t-\tau_2) - \\ &\quad \textstyle\int_{t-\tau_2}^{t} \left(\int_t^s u_{2s}(l-\tau_2)dl\right) ds\Big], \end{aligned} \tag{24}$$

and

$$\overline{\hat{\theta}}(t-\tau_2) = \theta(t-\tau_2) + \tau_2\overline{\omega}(t-\tau_2) - \int_{t-\tau_2}^{t} (s-t)u_{2s}(s-\tau_2)ds, \tag{25}$$

where $u_{1s}(\cdot), u_{2s}(\cdot)$ *are the functions defined respectively in (14) and (13), is globally uniformly asymptotically and locally exponentially stable.*

Proof. It follows from (8) that, for all $t \geq 2\tau_1$,

$$\begin{aligned} x_1(t) &= x_1(t-\tau_1) + \textstyle\int_{t-\tau_1}^{t} \dot{x}_1(s)ds \\ &= x_1(t-\tau_1) + \textstyle\int_{t-\tau_1}^{t} x_2(s)ds \\ &= x_1(t-\tau_1) + \tau_1 x_2(t) + \textstyle\int_{t-\tau_1}^{t} (x_2(s) - x_2(t))\, ds \\ &= x_1(t-\tau_1) + \tau_1 x_2(t) + \textstyle\int_{t-\tau_1}^{t} \left(\int_t^s \dot{x}_2(l)dl\right) ds \\ &= x_1(t-\tau_1) + \tau_1 x_2(t) \\ &\quad + \textstyle\int_{t-\tau_1}^{t} \left(\int_t^s u_{1s}(l-\tau_1)\sin\theta(l)dl\right) ds. \end{aligned}$$

Hence we obtain that the equality

$$x_2(t) = \overline{x}_2(t), \tag{26}$$

holds for all $t \geq 2\tau_1$. Similarly, it follows from (9) that, for all $t \geq 2\tau_1$,

$$y_2(t) = \overline{y}_2(t) \tag{27}$$

and from (10) that, for all $t \geq 2\tau_2$,

$$\omega(t) = \overline{\omega}(t). \tag{28}$$

It follows readily that, for all $t \geq 2(\tau_2 + \tau_1)$, the control laws (23), (22) are equal to the control laws (11), (12) used in Theorem 2. This concludes the proof.

6 Simulation Results

We have performed simulations for the system (8), (9) and (10) in closed-loop with the control laws (11) and (12) where the variables $x_2(t), y_2(t)$ and $\omega(t)$ are substituted by the right hand side of (26), (27) and (28) respectively. The initial conditions we have chosen are: $x_1(0) = x_2(0) = 0.5$, $\theta(0) = \omega(0) = 0.55$, $y_1(0) = y_2(0) = 1$. The behavior of the six state variables and the two control inputs is presented below

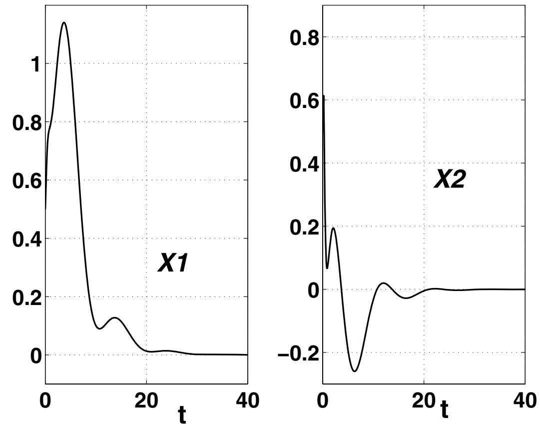

Fig. 1. Horizontal position and velocity

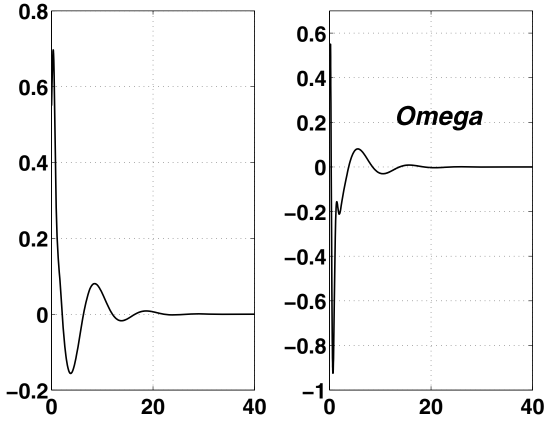

Fig. 2. Roll angle and velocity

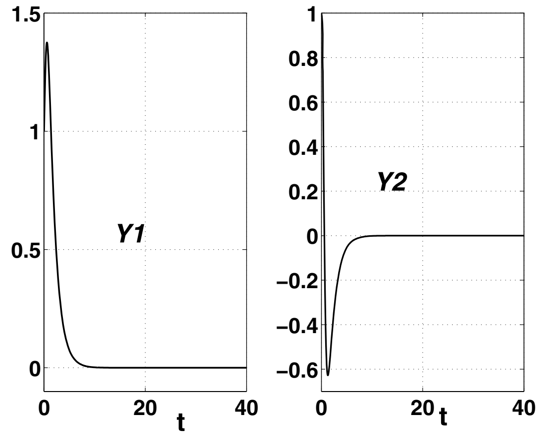

Fig. 3. Vertical position and velocity

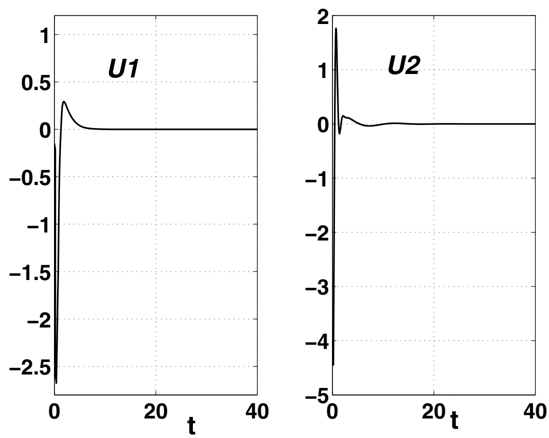

Fig. 4. Thrust and angular acceleration

7 Conclusion

In this work, two problems have been solved. First, we have achieved the global uniform asymptotic and local exponential stabilization of an aircraft PVTOL model with two delays in the inputs, using bounded state feedbacks. In a second step, we have shown how the presence of delays in the inputs can be exploited to achieve the global uniform asymptotic and local exponential stabilization of an aircraft PVTOL model when the variables of velocity are not measured. The main interest of the work is that it illustrates the possibility of applying recent theoretical results for nonlinear systems with delay to a physical system, very relevant from a practical point of view. Much remains to be done. We plan to study the following problems: Investigating whether or not there are possible ways to modify our construction in such a way that the resulting control laws are without distributed terms, determining control laws for the PVTOL with delay using not the forwarding approach but the backstepping approach, extending our results to the case where the exact values of the delay are unknown.

Acknowledgements. This work was supported by Conacyt, Mexico, Project 41276-Y and Grant 158335.

References

1. Do D.K. , Jiang Z.P., Pan J. (2003) On Global Tracking Control of a Aircraft Without Velocity Measurement. IEEE Transactions on Automatic Control 48:2212–2217
2. Hauser J., Sastry S., Meyer G. (1992) Nonlinear Control Design for Slightly Nonminimum Phase Systems: Application to V/STOL Aircraft. Automatica 28:665–679
3. Kharitonov V.L., Niculescu S.-I., Moreno J., Michiels W. (2003) Some remarks on static output feedback stabilization problem: necessary conditions for multiple delay controllers. In Proc. ECC 2003, Cambridge, U.K
4. Lin F., Zhang W., Brandt R.D. (1999), Robust Hovering Control of a PVTOL Aircraft. IEEE Transactions on Control Systems Technology 7:343–351
5. Marconi L., Isidori A. (2000) Robust global stabilization of a class of uncertain feedforward nonlinear systems. Systems & Control Letters 41:281–290
6. Marconi L., Isidori A., Serrani A (2002) Autonomous vertical landing on an oscillating platform: an internal-model based approach. Automatica 38:21–32
7. Martin P., Devasia S., Paden B. (1996) A Different Look at Output Tracking: Control of a VTOL aircraft. Automatica 32:101–107
8. Mazenc F., Praly L. (1996) Adding an integration and global asymptotic stabilization of feedforward systems. IEEE Transactions on Automatic Control 41:559–1578
9. Mazenc F., Bowong S. (2003) Tracking trajectories of the cart-pendulum system. Automatica 39:677–684
10. Mazenc F., Mondié S., Niculescu S.-I. (2003) Global Asymptotic Stabilization for Chains of Integrators with a Delay in the Input. IEEE Transactions Automatic Control 48:57–63
11. Mazenc F., Mondié S., Niculescu S.-I. (2004) Global Stabilization of Oscillators With Bounded Delayed Input. Systems & Control Letters 53:415–422
12. Mazenc F., Mondié S., Francisco R. (2004) Global Asymptotic Stabilization of Feedforward System with Delay in the Input. IEEE Transactions Automatic Control 49:844–850
13. Mazenc F., Francisco R., Mondié S. (2004) Global Asymptotic Stabilization of Feedforward System with Delay in the Input and vanishing perturbations. In Proc. 3rd IFAC Symposium on Systems, Structure and Control, pp. 216–221, Oaxaca, Mexico
14. Michiels W., Roose D. (2001) Global stabilization of multiple integrators with time-delay and input constraints. In Proc. IFAC Workshop on Time Delay Systems, pp. 266-271, Santa Fe, New Mexico
15. Palomino A., Castillo P., Fantoni I., Lozano R., Pégard C. (2003) Control Strategy Using Vision for the Stabilization of an Experimental PVTOL Aircraft Setup. In Proc. 42th IEEE Conference on Decision and Control, Maui, Hawaii

16. Sepulchre R., Janković M., Kokotovic P.V. (1997) Constructive Nonlinear Control, Springer-Verlag, London
17. Teel A.R. (1992) Global stabilization and restricted tracking for multiple integrators with bounded controls. Systems & Control Letters 18:165-171
18. Teel A.R. (1996) A Nonlinear Small Gain Theorem for the Analysis of Control Systems with Saturation. IEEE Transactions on Automatic Control 41:1256-1270
19. Tsinias J., Tzamtzi M.P. (2001) An explicit formula of bounded feedback stabilizers for feedforward systems, Systems & Control Letters 43:247-261

A Proof of Lemma 1

The fact that $\tau_2 \geq \tau_1$ ensures that the control law $u_{1s}(\cdot)$ defined in (11) is well defined. Due to the feedforward structure of the system (8), (9), (10), it is clear that the trajectories of this system in closed- loop with the bounded feedbacks (15), (12) are defined for all $t \geq 0$ (observe in particular that the finite escape time phenomenon obviously does not occur).

The next step of the proof consists in showing that $u_{2s}(\cdot)$ defined in (12) ensures that $|\theta(t)| \leq \frac{\pi}{4}$ when t is large enough. This proof is lengthy but simple.

First observe that

$$\begin{aligned} \omega(t-\tau_2) - \omega(t) &= \int_t^{t-\tau_2} \dot{\omega}(s)ds \\ &= \int_{t-\tau_2}^{t} \frac{L}{Mk^4}\sigma_4(\cdot)ds. \end{aligned} \tag{29}$$

It follows that

$$\dot{\omega} = -\frac{L}{Mk^4}\sigma_4(k^3\frac{M}{L}\omega(t) + \mu_1(t)) \tag{30}$$

where $\mu_1(t) = k^3\frac{M}{L}(\omega(t-\tau_2) - \omega(t)) + \sigma_3(\cdot)$ is a function such that, for all $t \geq 0$,

$$|\mu_1(t)| \leq \frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3. \tag{31}$$

The derivative of the positive definite and radially unbounded function

$$V_1(\omega) = \frac{1}{2}\omega^2 \tag{32}$$

along the trajectories of (30) satisfies

$$\begin{aligned} \dot{V}_1 &\leq -\frac{L}{Mk^4}\omega(t)\sigma_4(k^3\frac{M}{L}\omega(t) + \mu_1(t)) \\ &\leq -\frac{L}{Mk^4}|\omega(t)|\varepsilon_4\sigma(\frac{1}{\varepsilon_4}(k^3\frac{M}{L}|\omega(t)| - \frac{\tau_2}{k}\varepsilon_4 - \varepsilon_3)). \end{aligned} \tag{33}$$

It follows that, when $|\omega(t)| \geq 2\frac{L}{Mk^3}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)$,

$$\dot{V}_1 \leq -2\frac{L^2\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)\varepsilon_4}{M^2k^7}\sigma(\frac{1}{\varepsilon_4}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)) < 0. \tag{34}$$

It follows that there exists $T_1 \geq 0$ such that, for all $t \geq T_1$,

$$|\omega(t)| \leq 2\frac{L}{Mk^3}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right). \tag{35}$$

Combining (31) and (35), we deduce that, for all $t \geq T_1$,

$$\frac{1}{\varepsilon_4}\left|k^3\frac{M}{L}\omega(t)+\mu_1(t)\right| \leq \left(3\frac{\tau_2}{k}+3\frac{\varepsilon_3}{\varepsilon_4}\right) \leq \frac{1}{2}. \tag{36}$$

We deduce that there exists $T_2 \geq T_1$ such that, for all $t \geq T_2$,

$$\begin{aligned}\dot{\omega} &= -\tfrac{L}{Mk^4}k^3\tfrac{M}{L}\omega(t-\tau_2) - \tfrac{L}{Mk^4}\sigma_3(\cdot)\\ &= -\tfrac{1}{k}\omega(t-\tau_2) - \tfrac{L}{Mk^4}\sigma_3(\cdot).\end{aligned} \tag{37}$$

It follows that the derivative of the variable

$$\gamma = k\omega + \theta \tag{38}$$

satisfies

$$\begin{aligned}\dot{\gamma} &= \omega(t) - \omega(t-\tau_2) - \tfrac{L}{Mk^3}\sigma_3(k^3\tfrac{M}{L}\omega(t-\tau_2) + k^2\tfrac{M}{L}\theta(t-\tau_2) + \sigma_2(\cdot))\\ &= -\tfrac{L}{Mk^3}\sigma_3(k^3\tfrac{M}{L}\omega(t) + k^2\tfrac{M}{L}\theta(t) + \mu_2(t)) + \mu_3(t)\\ &= -\tfrac{L}{Mk^3}\sigma_3(\tfrac{M}{L}k^2\gamma(t) + \mu_2(t)) + \mu_3(t),\end{aligned} \tag{39}$$

where $\mu_2(t)$ and $\mu_3(t)$ are continuous functions such that

$$\begin{aligned}&|\mu_2(t)| \leq k^3\tfrac{M}{L}\left|\omega(t)-\omega(t-\tau_2)\right| + k^2\tfrac{M}{L}\left|\theta(t)-\theta(t-\tau_2)\right| + \varepsilon_2,\\ &|\mu_3(t)| \leq \left|\omega(t)-\omega(t-\tau_2)\right|.\end{aligned} \tag{40}$$

From (29) and (35), we deduce that there exists $T_3 \geq T_2$ such that, for all $t \geq T_3$,

$$\begin{aligned}|\mu_3(t)| &\leq \tfrac{\tau_2 L\varepsilon_4}{Mk^4},\\ |\mu_2(t)| &\leq k^3\tfrac{M}{L}\tfrac{\tau_2 L\varepsilon_4}{Mk^4} + \varepsilon_2 + k^2\tfrac{M}{L}\int_{t-\tau_2}^{t}|\omega(s)|\,ds\\ &\leq \varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + \left(\tfrac{\tau_2}{k}+\tfrac{2\tau_2^2}{k^2}\right)\varepsilon_4.\end{aligned}$$

It follows that the derivative of the positive definite and radially unbounded function

$$V_2(\gamma) = \frac{1}{2}\frac{Mk^3}{L}\gamma^2 \tag{41}$$

along the trajectories of (39) satisfies, when $t \geq T_3$

$$\begin{aligned}\dot{V}_2 &\leq -\tfrac{Mk^3}{L}\tfrac{L}{Mk^3}\gamma(t)\sigma_3(\tfrac{M}{L}k^2\gamma(t) + \mu_2(t)) + \gamma\tfrac{Mk^3}{L}\mu_3(t)\\ &\leq -\varepsilon_3|\gamma(t)|\sigma(\tfrac{1}{\varepsilon_3}(\tfrac{M}{L}k^2|\gamma(t)| - (\varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + (\tfrac{\tau_2}{k}+\tfrac{2\tau_2^2}{k^2})\varepsilon_4)))) + \tfrac{\tau_2\varepsilon_4}{k}|\gamma(t)|.\end{aligned} \tag{42}$$

It follows that when $t \geq T_3$ and when

$$|\gamma(t)| \geq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k}+\frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right),$$

the inequality

$$\dot{V}_2 \leq -\varepsilon_3|\gamma(t)|\sigma\left(\tfrac{1}{\varepsilon_3}\left(\varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + \left(\tfrac{\tau_2}{k} + \tfrac{2\tau_2^2}{k^2}\right)\varepsilon_4\right)\right) + \tfrac{\tau_2\varepsilon_4}{k}|\gamma(t)| \tag{43}$$

is satisfied. The values of the parameters present in this inequality and the properties of $\sigma(\cdot)$ imply that when $t \geq T_3$ and when

$$|\gamma(t)| \geq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right),$$

the following inequality is satisfied.

$$\begin{aligned} \dot{V}_2 &\leq -|\gamma(t)|\left(\varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + \left(\tfrac{\tau_2}{k} + \tfrac{2\tau_2^2}{k^2}\right)\varepsilon_4\right) + \tfrac{\tau_2\varepsilon_4}{k}|\gamma(t)| \\ &\leq -|\gamma(t)|\left(\varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + \tfrac{2\tau_2^2}{k^2}\varepsilon_4\right) < 0. \end{aligned} \tag{44}$$

We deduce that there exists $T_4 \geq T_3$ such that, for all $t \geq T_4$,

$$|\gamma(t)| \leq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right). \tag{45}$$

This inequality, the definition of γ (see (38)) and (35) imply that, when $t \geq T_4$,

$$\begin{aligned} |\theta(t)| &\leq 2\tfrac{L}{Mk^2}\left(\tfrac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right) \\ &\quad + 2\tfrac{L}{Mk^2}\left(\varepsilon_2 + \tfrac{2\tau_2}{k}\varepsilon_3 + \left(\tfrac{\tau_2}{k} + \tfrac{2\tau_2^2}{k^2}\right)\varepsilon_4\right) \\ &\leq \tfrac{\pi}{4}. \end{aligned}$$

The result is proved.