Global Uniform Asymptotic Stabilization of an Underactuated Surface Vessel: Experimental Results

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Abstract—In a recent work, explicit formulas of smooth time-varying state feedbacks which make the origin of a model of an underactuated surface vessel globally uniformly asymptotically stable are proposed. In the present work experimental results are presented where these control laws are implemented for control of a model of an offshore supply vessel in order to investigate the values and limitations of the theoretical results. In the experiments the ship is exposed to perturbations including unmodeled dynamics, waves, currents and measurement noise. The experiments indicate that the control system possesses some robustness to these perturbations, something which complies with theory. The experimental results suggest that unmodeled ship dynamics and environmental disturbances should be taken into account in the controller design for practical implementation.

Index Terms—Experimental results, time-varying feedback, underactuated systems, uniform asymptotic stability.

I. INTRODUCTION

DYNAMIC positioning of surface vessels is required in many offshore oil field operations such as drilling, pipe-laying and diving support. Critical to the success of a dynamically positioned surface vessel is its capability for accurate and reliable control, subject to environmental disturbances as well as to configuration related changes, such as a reduced number of available control inputs. This reduction may be the result of an actuator failure or a deliberate decision to limit the number of actuators due to e.g., cost and weight considerations.

In [24], we have considered the dynamic positioning control problem for a ship that has no side thruster, but two independent main thrusters located at a distance from the center line in order to provide both surge force and yaw moment. The control problem considered in this paper was to find a control law that stabilized both the position variables and the orientation, using only the two available controls. As open-loop control does not compensate for disturbances and model errors, we have constructed a family of globally uniformly asymptotically stabilizing feedback control laws. Since the proposed feedback control three degrees of freedom with only two independent controls, we have, hence, solved an underactuated control problem.

Control of underactuated systems is a continuation of the research on nonholonomic systems. While the nonholonomic systems have constraints on the velocity, underactuation leads to constraints on the acceleration. Consequently, for underactuated systems we have to consider the dynamics in addition to the kinematics of the system in the control design. When the dynamics is included, the systems have a drift vector field and this poses new demands on the controllability analysis and the control design as compared to driftless systems. In recent years, nonholonomic systems have been a topic of much interest in the control society. Control of nonholonomic systems has proved to be a challenging problem, inherently nonlinear and not amenable to linear control theory. For the stabilization of nonholonomic systems which do not satisfy the conditions of Brockett [5], several approaches have been proposed. A review of nonholonomic systems control is given in [20]. To mention a few, stabilization of equilibrium manifolds and the use of discontinuous control was proposed in [4] and [3] while [30] was the first to show how continuous time-varying feedback laws could asymptotically stabilize nonholonomic systems, in particular a nonholonomic cart.

Control of underactuated ships is an active topic of research. Underactuated tracking control of ships has been considered for instance in [18], [15], [1], [27], [31], [6], and [11]. Concerning the stabilization problem, it is seen from results by [5], [9] and [33] that the ship is not even locally asymptotically stabilizable by continuous static state feedback. However, the surface vessel is still locally strongly observable and small time locally controllable [25], and by [10] the ship is then locally asymptotically stabilizable in small time by means of an almost smooth periodic time-varying feedback law. However, since the underactuated ship is not a controllable driftless system, the results of [8], [10] do not allow us to claim that this system is globally asymptotically stabilizable by time-varying feedbacks.

For the stabilization of the underactuated ship, in [32] a continuous feedback control law is proposed that instead asymptotically stabilizes an equilibrium manifold. The desired equilibrium point is then stable as all the system variables are bounded by the initial conditions of the system. Furthermore, the position variables with this approach converge exponentially to their desired values. The course angle however converges to some constant value, but not necessarily to zero. In [29] a discontinuous feedback control law is proposed, and this provides exponential convergence to the desired equilibrium point, under certain assumptions on the initial value. In [25] a time-varying feedback control law is proposed that provides exponential (with respect
to a given dilation) stability of the desired equilibrium point. However the feedback law only locally stabilizes the desired equilibrium point, and the size of the region of attraction is not known. In [27] a time-varying feedback control law is proposed that provides semiglobal practical exponential stability of a simplified model of the ship, where the surge and yaw velocities are considered as controls. In [7] a geometric framework for controllability analysis and motion control is proposed for mechanical systems on Lie groups, including the underactuated hovercraft. In [24], the full ship model including both the dynamics and the kinematics, with surge force and yaw moment controls are considered, and a globally uniformly stabilizing feedback control law was constructed.

Although there has been a lot of research on the topic of underactuated systems, quite few experimental results have been reported. In this paper we present experimental results with the aim of understanding more about the value and the limitations of the theory presented in [24]. The feedback law is implemented for control of a model of an offshore supply vessel, scale 1:70, moving in a pool at the Guidance, Navigation and Control laboratory, NTNU.

The work is organized as follows. The ship model is presented in Section II. In Section III, the feedback control law is given and in Section IV the experimental results are presented.

II. SHIP MODEL

For the development of the control law, we use a nonlinear model of the ship based on [14]. The dynamic equations of the ship are

\[
\begin{align*}
\dot{u} &= \frac{m_{11} m_{12}}{m_{11} m_{22}} v - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_1 \\
\dot{v} &= -\frac{m_{11} m_{12}}{m_{22}} - \frac{d_{12}}{m_{22}} v \\
\dot{r} &= \frac{m_{11} m_{22}}{m_{33}} - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 .
\end{align*}
\]

The variables \( u, v \) and \( r \) are the velocities in surge, sway and yaw, respectively, see Fig. 1. The parameters \( m_{11}, m_{22} > 0 \) are given by the ship inertia and added mass effects. The parameters \( d_{11}, d_{12}, d_{33} > 0 \) are given by the hydrodynamic damping. The available controls are the surge control force \( \tau_1 \), and the yaw control moment \( \tau_2 \). We do not, however, have an available control in sway, and the problem of controlling the ship in three degrees of freedom is therefore an underactuated control problem. When modeling the ship, the dynamics associated with the motion in heave, roll and pitch are assumed to be negligible. It is furthermore assumed that the inertia and damping matrices are diagonal. This is true for ships having port/starboard and fore/aft symmetry. Most ships have port/starboard symmetry. Non-symmetry fore/ast of the ship implies that the off-diagonal terms of the inertia matrix \( m_{23} \neq 0 \) and \( m_{32} \neq 0 \), and also for the damping matrix \( d_{23} \neq 0 \) and \( d_{32} \neq 0 \). These off-diagonal terms will, however, be small compared to the diagonal elements \( m_{ii} \) and \( d_{ii} \) for most ships. Non-symmetry fore/ast will also give some extra cross-terms due to Coriolis and centripetal forces. Control design in the general case where also the off-diagonal terms are taken into account, is trivial to solve for a fully actuated ship while it is still a topic of future research for the underactuated ship.

The kinematics of the ship are described by

\[
\begin{align*}
\dot{x} &= \cos(\psi)u - \sin(\psi)v \\
\dot{y} &= \sin(\psi)u + \cos(\psi)v \\
\dot{\psi} &= r
\end{align*}
\]

where \( x, y, \) and \( \psi \) give the position and orientation of the ship in the Earth-fixed frame. To obtain simpler, polynomial equations we use the same global coordinate transformation as in [25]

\[
\begin{align*}
z_1 &= \cos(\psi)x + \sin(\psi)y \\
z_2 &= -\sin(\psi)x + \cos(\psi)y \\
z_3 &= \psi .
\end{align*}
\]

The resulting model of the ship is then

\[
\begin{align*}
\dot{z}_1 &= u + z_2 r \\
\dot{z}_2 &= v - z_1 r \\
\dot{z}_3 &= r \\
\dot{u} &= -\frac{m_{11} m_{12}}{m_{11} m_{22}} u - \frac{d_{12}}{m_{22}} v - \frac{1}{m_{11}} \tau_1 \\
\dot{v} &= -\frac{m_{11} m_{12}}{m_{33}} - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 .
\end{align*}
\]

III. CONTROL LAW

In order to transform the model into a form that is suitable for applying the results in [24], we use the following coordinate and input transformations:

\[
\begin{align*}
Z_2 &= z_2 + \frac{m_{22}}{d_{22}} v \\
\mu &= -\frac{m_{11}}{d_{22}} u - z_1 \\
\tau_\mu &= \frac{d_{22} - d_{11}}{m_{11}} - \frac{m_{11} m_{12}}{m_{22}} \mu - Z_2 r - \frac{1}{d_{22}} \tau_1 \\
\tau_r &= \frac{m_{11} m_{22} - d_{33}}{m_{33}} r - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 .
\end{align*}
\]
(The details of this choice of transformations can be found in [24].) The resulting model equations are

\[
\begin{align*}
\dot{z}_1 &= -\frac{d}{c}z_1 - \frac{d}{c}\mu + Z_2 r - \frac{d}{c}r \\
\dot{Z}_2 &= \mu r \\
\dot{\theta} &= -d\nu + d(z_1 + \mu)r \\
\dot{\mu} &= \tau_\mu \\
\dot{r} &= \tau_r
\end{align*}
\]  

where \( c = m_{11}/m_{22}, d = d_{12}/m_{22} \). Based on this model the following result can be shown:

**Theorem 1:** Let

\[
Z_3 = z_3 + k_2 \cos(\omega t) Z_2
\]

and \( k_2, k_3, \mu, r, \omega \) be strictly positive parameters such that

\[
k_2 \leq 1, \; \omega \geq k_3.
\]

Consider the transformed ship dynamics (15). Then the origin of the system is globally uniformly asymptotically stabilized by the feedbacks

\[
\begin{align*}
\tau_\mu &= -k_\mu (\mu - \mu_f) + \mu_f - \lambda [Z_2 + 2Z_3 k_2 \cos(\omega t)] r \\
\tau_r &= -k_r (r - r_f) + r_f - \lambda [Z_2 \mu_f + 2Z_3 + 2Z_3 k_2 \cos(\omega t) \mu_f]
\end{align*}
\]  

where

\[
\begin{align*}
V_1(Z_2, Z_3) &= Z_2^2 + 2Z_3^2 \\
\lambda &= 2 + \frac{k_3}{3} - \frac{k_3 \sin(2t)}{6} (1 + V_1)^2 \\
\mu_f &= -\frac{\sin(\omega t) Z_2^2}{2(0.001 + Z_2^2)} \\
r_f &= -k_3 Z_3 + k_2 \omega \sin(\omega t) Z_2 \\
\end{align*}
\]

Moreover, the origin of the system (15) is also globally uniformly asymptotically stabilized by the feedbacks

\[
\begin{align*}
\tau_\mu &= -k_\mu (\mu - \mu_f) + \mu_f \\
\tau_r &= -k_r (r - r_f) + r_f
\end{align*}
\]  

**Proof:** The proof is given in [24].

**Remark 1:** We are able to prove global uniform asymptotic stability with the relatively simple control law (23) by using the results of [23] that allow us to find a strict Lyapunov function, together with the robust backstepping results of [17]. The controller (18) on the other hand is developed using standard backstepping [22], [21].

**Remark 2:** The controller (23) makes the transformed surge velocity \( \mu \) and the angular velocity \( r \) converge to the desired virtual inputs \( \mu_f \) and \( r_f \) of the subsystem \( (z_1, Z_2, z_3, \theta) \). The virtual inputs \( \mu_f \) and \( r_f \) are chosen to make the ship move somewhat forward and backward at the same time as it turns, in order to use the resulting Coriolis and centripetal forces together with the kinematics to make the ship move sideways in a manner that controls the unactuated state variables \( Z_2 \) and \( \nu \) to zero. In addition the transformed surge velocity \( \mu \) is chosen to create a virtual control input \(-(d/c)z_1\) in the \( \dot{z}_1 \) equation to steer \( z_1 \) to zero.

IV. EXPERIMENTAL RESULTS

The experiments were performed at the Guidance, Navigation and Control Laboratory, NTNU. The laboratory includes a model ship, CyberShip I, which is a model of an offshore supply vessel, scale 1:70, see Fig. 2. The model ship has a mass of 17.6 kg, and a length of 1.19 m. The relationship between the speed of the ship and the model ship is

\[
U_S \approx 8.37U_M \left[ \frac{m}{s} \right]
\]

where the subscripts \( S \) and \( M \) denote the ship and the model, respectively.

The ship moves in a 6 \times 10 m pool. The position and orientation of the ship are monitored by two infrared cameras, see Fig. 3. The infrared cameras detect 3 markers mounted at the ship. The position of these markers are transmitted to a PC-386 computer where the ship position and orientation in Earth-fixed coordinates are calculated. These coordinates are transmitted to a dSPACE signal processor. This communicates over a dSPACE bus with a Pentium 166 MHz computer where the feedback control law is implemented. Note however, as there is no separation principle for nonlinear systems, this scheme does not guarantee stability even though the observer is proven to be stable and we have proven that the closed-loop system with full-state feedback is stable. The thruster commands are sent through the signal processor, by a radio transmitter, to the ship. The sampling frequency used in the experiments was 50 Hz.

Figs. 4–20 show the results of an experiment with the controller (23). It should be noted that in choosing the control parameters there is a lot of freedom. The choices made in the experiment presented here have a certain degree of arbitrariness, and we do not at all conjecture that this is the best or optimal choice in any sense. The control parameters have to satisfy (17).
thruster commands

radio transmitter

radio

stim con

Pentium 166 MHz

joystick

dspace signal processor

dspace bus

feedback control system implemented in Matlab simulink

Kinematic transformation from camera to world coordinates

Fig. 3. Experimental setup.

Fig. 4. Ship motion in the xy plane [m]. (a) Experiments. (b) Simulations.

Fig. 5. Ship position variable (-) together with its set point value (-) [m].
(a) Experiments. (b) Simulations.
in order for the theoretical result to hold. For practical implementation we have limitations on $\omega$. As $\omega$ is the frequency of the commanded controls, we have to choose this below the actuator rate limits given by the actuators used. Testing showed that $\omega = 0.1 \text{ [rad/s]}$ did not give rise to any significant actuator rate saturation. Having chosen $\omega$, secondly we choose $k_2$ and $k_3$. With the fixed choice of $\omega$, the larger we choose $k_2$ the larger we can choose $k_3$. The magnitude of $k_2$ and $k_3$ can in the analysis be seen to be factors in the convergence rates of $Z_2$ and $Z_3$, respectively. We therefore want to choose both $k_2$ and $k_3$ as close to their upper limits as possible, as long as we do not have severe actuator magnitude limitations. However, there is another factor affecting the choice of $k_2$ and $k_3$. Due to the underactuation we do not have direct control of $Z_2$, the transformed sway position variable. The yaw velocity $r$ is used together with the transformed surge velocity $\mu$ to create a force to control the sway variables. Choosing $k_3$ too high, the control law will keep the yaw angle $\psi = z_2$ quite close to zero, something which increases the necessary oscillations in surge to make the unactuated variables converge. Allowing $\psi$ to converge slower than $Z_2$ will therefore demand fewer oscillations in surge before the unactuated variables converge. Based on this, we choose $k_2$ equal to its upper limit, while we choose $k_3$ to be 50% of its upper limit. Finally, the parameters $k_\mu$ and $k_r$ are chosen. These parameters are factors in the convergence rates of $\mu$ and $r$ to the virtual control laws $\mu_f$ and $r_f$, respectively. Since $\mu$ and $r$ are virtual controls for the system

$$\begin{cases}
  \dot{z}_2 = \mu r \\
  \dot{z}_3 = r
\end{cases}$$

(25)

it is natural to seek convergence rates for $\mu$ and $r$ to $\mu_f$ and $r_f$ that are about a decade higher than the convergence rates of $Z_2$ and $Z_3$. Without a thorough convergence analysis, but based on the expressions for the Lyapunov functions and their derivatives,
we chose $k_\mu = k_r = 1$ which immediately gave a stable scheme without significant actuator magnitude saturation. To conclude, there is a lot of freedom in the choice of control parameters. Efforts should, hence, be made to find good criteria linking the control parameters to convergence rates and energy consumption, to guide this choice for practical implementation. As for the experiment presented in Figs. 4–20, the control parameters were chosen in accordance with the above discussion, and the set-point in the pool $x_d$, $y_d$ and $\psi_d$ was chosen as follows:

$$
\begin{align*}
\omega &= 0.1 \text{ m/s} \\
k_\mu &= 1 \\
k_r &= 1 \\
k_3 &= 0.05 \\
\psi_d &= 0 \text{ deg}
\end{align*}
$$

such that the variables $z_1$, $z_2$ and $z_3$ were given by

$$
\begin{align*}
z_1 &= \cos(\psi)(x - 5) + \sin(\psi)(y - 4) \\
z_2 &= -\sin(\psi)(x - 5) + \cos(\psi)(y - 4) \\
z_3 &= \psi
\end{align*}
$$

In all the experiments performed, including the one presented in Figs. 4–20, the ship converged to a neighborhood of the desired equilibrium point. Where in this neighborhood the ship was after the logging time of 300 s in each of the experiments was seen to depend on the initial conditions, the frequencies, magnitude and direction of the waves, the currents, and the measurement noise during the particular experiment. In addition to this, the camera system failed from time to time, in particular when the ship moved between sectors of the pool covered by...
different cameras. During a camera failure, the estimates started drifting such that the feedback became wrong, and when the camera system had to be reinitialized this implied a disturbance to the control system which is reflected by spikes in the time evolution of the system variables and controls. The number and duration of camera failures during an experiment also affected the convergence of the ship. The time evolution of the status of the camera system in the particular experiment presented here, is seen in Fig. 18. (The initial time of the experiments \( t_0 = 72 \) s, such that the time along the \( x \) axis of the figures is \( t' = t - 72 \) s).

The ship motion in the \( xy \) plane is shown in Fig. 4. In Figs. 5 and 6 we see the time evolution of the variables \( x \) and \( z_1 \), and Figs. 9 and 10 show the time evolution of the variables \( \psi \) and \( Z_3 \). We see that these variables oscillate, due to the oscillations in the velocities \( u \) (Fig. 11) and \( r \) (Fig. 14) generated by the control law in order to set up a force that makes the unactuated degrees of freedom converge. We can see that the ship oscillations are quite close to the oscillations demanded by the control system, as can be seen in Figs. 12 and 15 and are, hence, a result of the periodic time-varying control approach used. These oscillations will, hence, be sustained as long as there are disturbances driving the unactuated degree of freedom away from its reference value. The time evolutions of the unactuated variables \( y, Z_2 \) and \( v \) are seen in Figs. 7, 8 and 13. We see that the unactuated variables converge quite close to their desired values, but also there are some small deviations.

The commanded surge control force \( r_1 \) and the commanded yaw control moment \( r_3 \) are shown in Figs. 16–17. The actuator magnitude saturations of \( r_1 \) and \( r_3 \) are approximately 1 N and 1 Nm, respectively, so we see that the control system experienced some actuator saturation in \( r_1 \) initially.

The norms

\[
\rho_1 = \sqrt{x^2 + y^2 + \psi^2 + u^2 + v^2 + r^2} \tag{31}
\]

and

\[
\rho_2 = \sqrt{z_1^2 + Z_3^2 + (\mu - \mu_f)^2 + \nu^2 + (r - r_f)^2} \tag{32}
\]
are shown in Figs. 19–20. To better understand the relation between the theoretical and experimental results, we have performed simulations under the ideal conditions of a perfect ship model with no thruster saturation, no measurement noise and no environmental disturbances. The simulation model is (1)–(3), (4)–(6) with parameters

\[ m_{11} = 19.0 \quad d_{11} = 4.0 \quad (33) \]

\[ m_{22} = 35.2 \quad d_{22} = 10.0 \quad (34) \]

\[ m_{33} = 4.2 \quad d_{33} = 1.0. \quad (35) \]

The controller parameters and the set point of the simulations were chosen equal to (26). Furthermore, the initial conditions were chosen equal to the initial conditions in (30), and the initial time \( t_0 = 72 \) s, as in the experiment. The simulation results are showed together with the experimental results in Figs. 4–17 and Figs. 19–20, in order to provide a back-to-back comparison of the results of the numerical simulation and the experiment. Comparing the figures presenting the experiment with the numerical simulation results, we see that qualitatively the transient behavior is quite similar. In the experiments however, the ship converges to a neighborhood of the set point instead of converging to the set point itself. (We do not fully see in the figures that the ship converges to the origin in the simulations due to the limited time of 300 s of the experiments.) When it comes to the numeric values, we observe some differences also in the transient behavior. In Fig. 4 we see that the turns the ship makes in the experiment are not as sharp as those in the simulations. Since the kinematic model is well-known, it is probable that the differences between experimental and simulation results are due to effects not taken into account in the dynamic model. Having a closer look at the dynamic variables, we see in Figs. 11–12 that the surge velocity \( u \) and the transformed surge velocity \( \mu \) are not able to follow their references as well in the experiments as in the simulations. The same applies for the angular velocity shown in Figs. 14–15. Since making \( \mu \) and \( r \) follow the virtual control inputs \( \mu_f \) and \( r_f \) is at the core of the control design (in order to move forward and backward together with
turning to create the necessary control force in the unactuated sway dynamics), it is therefore to be expected that neither the sway velocity $v$ converges as well as in the simulations, Fig. 13. A question is which perturbations of the dynamic model that give this result. There are a number of perturbations that the ship possibly was subjected to during the experiments, and that were not taken into account in the ideal model that the control design and analysis is based on, and which is used for simulations. There were waves, currents, and also hydrodynamic effects as the ship moved close to the sides of the pool. There may also have been modeling errors both in model structure and parameters of Cybership I (see Section II), there were measurement noise and thruster limitations (magnitude and rate). Also, there may have been small errors in the feedback since we used estimates of the position and velocity from a nonlinear observer. To seek to identify which of these factors that may have been the most important, we performed simulations where we introduced different perturbations to the ideal dynamic model. Waves and currents, model parameter errors and unmodeled inertia and damping cross terms, measurement noise and thruster magnitude limitations (the control frequency $\omega$ of the experiments was so low that we do not expect thruster rate limitations to be a problem) were introduced.

Cybership I has actuator saturation in the magnitude of 1.0 – 1.2 N and 1.0 – 1.2 Nm in surge control force and yaw control moment, respectively. Introducing actuator magnitude saturation at 1.0 N and 1.0 Nm, the transient values of $u$ were lowered to values quite similar to those in the experiment. This may imply that the quantitative differences in the transient values of $u$ may be the result of actuator magnitude saturation. There were however no change in the transient value of $r$, which was as expected since we see in Fig. 17 that the commanded yaw control moment is below the magnitude saturation in both experiments and simulations.

We then performed simulations with measurement noise (without actuator saturation). We added measurement noise of zero mean value and standard deviation 5 cm (corresponding to 3.5 m for the full scale off-

Fig. 14. Angular velocity $r$ (-) together with its set point value (-) [deg/s]. (a) Experiments. (b) Simulations.

Fig. 15. Time evolution of the error $r - r_f$ [deg/s]. (a) Experiments. (b) Simulations.
shore supply vessel) to the position variables $x$ and $y$, and zero mean value noise with a standard deviation of 1 deg to the yaw angle $\psi$. This corresponds to the maximum measurement noise a supply vessel typically could experience. The controls then of course became more noisy, something which was transferred to the states, mainly to the surge and sway velocities $u$ and $r$. This agrees with the fact that it is the dynamic equations of these two states that are directly affected by $\tau_1$ and $\tau_3$, while for the other states the noise in the controls is mainly filtered away by the dynamics and kinematics. Despite the quite severe measurement noise we add in the simulation, the main features of the simulation results remain the same. Comparing these results with the experiment, we can see from Figs. 11 and 14 that $u$ and $r$ are somewhat affected by noise, but well below the noise added in the simulations. Measurement noise is therefore unlikely to be a main reason for the differences in experimental and simulation results.

We then removed both actuator saturation and measurement noise, and changed the simulation model parameters by adding 20% to the diagonal elements of the inertia and added mass matrix, $m_{ii}$, $i = 1, 2, 3$, and by subtracting 10% from the diagonal elements of the damping matrix, $d_{ii}$, $i = 1, 2, 3$, i.e., making the ship inertia larger and the stabilizing hydrodynamic damping less than the controller is designed for, to investigate whether model parameter errors can be a source for differences between experiment and simulation results. This gave rise to some small deformation in the earlier quite sinus like time evolution of the position variable $x$ and also gave some small ripples in the other states. The simulation results, thus, got somewhat closer to the experiment results, and this indicates that there may have been model parameter errors in the experiments, but these do not seem to be a major reason for the differences in experiment and simulation results. To further investigate possible modeling errors, we included in addition to the parameter errors of the diagonal terms the nondiagonal terms $m_{23}$ and $m_{32}$ in the matrix of inertia and added mass, and also $d_{23}$ and $d_{32}$ in the damping matrix. These terms where chosen to be 30% of the current values.
source of the steady oscillations that the system showed in the experiments. To conclude, the simulations indicated that there were two main reasons for the differences in simulation and experiment results. Modeling errors, in particular unmodeled dynamics, was seen to be important to the difference in transient behavior, and environmental disturbances was seen to create stationary oscillations as observed in the experiments.

The simulations and experiments furthermore illustrated that the controlled ship had some robustness properties, something which complies with the theoretical results in [24]. In particular, since the origin is showed to be globally uniformly asymptotically stable, the ship has some robustness properties (that are not guaranteed by global asymptotic stability). In particular, when the ship is exposed to small perturbations it will converge to a neighborhood of the origin (small-signal $L_\infty$ stability). The size of this neighborhood is strictly increasing with the size of the perturbations [19, Lemma 9.3]. During the experiments the ship was subjected to a number of perturbations, and also in the simulations. There were modeling errors both in model structure of $m_{22}$, $m_{33}$, $d_{22}$ and $d_{33}$, respectively. Although the choice of these terms had to be somewhat random, the simulated behavior showed a closer resemblance with the results of the experiment. In short the time evolution of the states was less smooth and the turns of the ship were seen to be less sharp in the $xy$ plane. This indicates that the simplifying assumptions of diagonal matrices of inertia and hydrodynamic damping may be an important reason for the differences seen between experimental and theoretical results.

Removing the modeling errors, we then introduced an environmental disturbance in the simulation model. A constant earth/fixed force was used as a model for the bias of currents and waves that affected the ship. We exposed the ship to a constant force of 0.05 N in both the Earth-fixed $x$- and $y$-directions. This also gave a yaw moment as the body-fixed coordinate system was placed 40 cm in front of the center of gravity. This did not affect the transient behavior as much as the "steady-state" behavior. Instead of converging toward zero as in the simulation under ideal assumptions, now the unactuated sway position variable $y$ and sway velocity $v$ after about 150 s kept oscillating with a small amplitude about an error of about 20 cm. One of the reasons for this may be that the actuated velocities $u$ and $r$ are not able to follow the virtual control functions $w_f$ and $r_f$ as well as without the disturbance. Another reason may be the lack of disturbance adaptation or integral effects in $y$. Furthermore, we saw that the actuated velocities $u$ and $r$ went into steady oscillations about zero. This agrees with the control development where $u$ and $r$ are used as virtual inputs to the unactuated system. When the unactuated variables $y$ and $v$ are not zero, the controller creates oscillations in the virtual inputs $u$ and $r$ to create a control force in sway. Therefore, when due to disturbances $y$ and $v$ do not converge to zero, the control algorithm will create steady oscillations in $u$ and $r$. This is also reflected in the controls $t_1$ and $t_2$. A similar behavior is also seen in the experiments. In the time evolution of $u$ and $v$ during the experiment, shown in Figs. 11 and 14, we see corresponding oscillations, and this is also reflected in the controls, Figs. 16 and 17. These simulations may indicate that environmental disturbances may be a major

Fig. 18. Status of the camera system during the experiment. Value 3: OK. Value 0: failure.

Fig. 19. Norm $p_1$. (a) Experiments. (b) Simulations.
and parameters of Cybership I (see Section II). Furthermore, there were measurement noise, thruster limitations (magnitude and rate), waves, currents, and also hydrodynamic effects as the ship moved close to the sides of the pool, that were not taken into account in the control design and analysis. Also, there may have been small errors in the feedback since we used estimates of the position and velocity from a nonlinear observer. Simulations showed that without all these perturbations, the ship converged to the origin. The magnitude of the perturbations of the ship in the experiments could not be measured, but evidently they were small enough for the robustness result to hold.

For comparison, in order to further understand the action of the relatively simple control law (23), we also performed experiments with \( \tau_u \) and \( \tau_r \) given by the full backstepping control law (18). The control law (18) aims at cancelling undesired terms, as opposed to the simpler control law in (23) that aims at rather dominating the undesired terms. Clearly we would expect the latter type of feedback in addition to being simpler also to be more robust. This hypothesis was supported by the experimental results; Even though one could expect that the controller gains \( k_u \) and \( k_r \) in (23) should have to be chosen larger than the gains \( k_u \) and \( k_r \) in (18), since (23) aims at dominating the undesired terms rather than cancelling them, the experiments showed the opposite. In particular, in the experiments with the control law (18) we had to increase \( k_u \) and \( k_r \) from 1 (used in the experiments with (23)) to 2 in order to avoid actuator saturation and get good performance. A probable reason for this is that the attempts of (18) to cancel system dynamics, due to modeling errors rather introduced some destabilizing terms and, hence, the controller gains \( k_u \) and \( k_r \) had to be chosen large enough to dominate these. Furthermore, it was seen in the experiments that despite more control efforts, the stationary errors were generally larger than with the controller (23). This indicates that (23) gives a system that is more robust to modeling errors than (18) does.

V. CONCLUSION

In this paper we have presented experimental results in order to better understand the value and the limitations of the theory presented in [24]. The globally asymptotically uniformly stabilizing control law presented in [24] was implemented for the control of a model of an offshore supply vessel, scale 1:70, at the GNC laboratory, NTNU. The experimental results complied with the robustness properties given by the theoretical results. In particular, since the origin was proved to be a globally uniformly asymptotically stable equilibrium point of the ship, when the ship was subjected to perturbations like unmodeled dynamics, waves, currents and measurement noise, theory predicted that the ship would converge to a neighborhood of the origin. This was verified in all the experiments performed, as the ship was seen to converge to a neighborhood of the origin. The ship went into stationary oscillations in this neighborhood. Specifically, while the unactuated sway variables were kept quite close to their reference values, there were stationary oscillations in the actuated degrees of freedom, i.e., in surge and yaw. Simulations indicated that the main reason for these stationary oscillations was environmental disturbances. This suggests that future work should aim at taking into account these disturbances, possibly including an adaptation scheme. It is still an open question whether it is possible to fully adapt and counteract disturbances in all three degrees of freedom having only two independent control inputs. (Preliminary results are given in [13], [28], [2] and [12].) Furthermore comparing experiments with simulations it was seen that actuator saturation could have some, but probably not an important impact on the transient behavior in surge. Unmodeled dynamics, however, seemed to be quite important for the differences in transient behavior. In particular, in future work the nondiagonal elements of the inertia and damping matrices should be taken into account in the control design and analysis.

The experiments supported the conjecture that a simplified version of the control law aiming at dominating undesired terms rather than cancelling them, will have better robustness to modeling errors than a full backstepping control law. It was also noted that there was a lot of freedom in choosing the control parameters, and efforts should be made to find good criteria for this choice for practical implementation.
REFERENCES


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